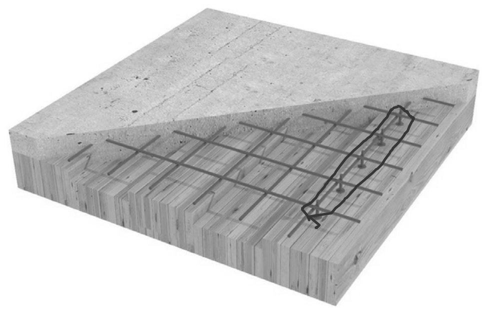
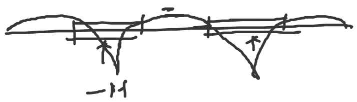
Architecture 544
Wood Structures

# Wood Composite Systems





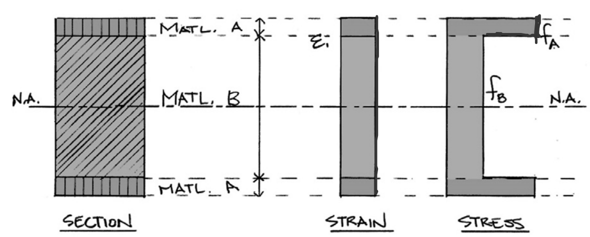
- Strain Compatibility
- Transformed Sections
- Flitched Beams
- Steel Sandwiched Beams
- Wood – Concrete Composites



## Strain Compatibility

With two materials bonded together, both will act as one, and the deformation in each is the same.

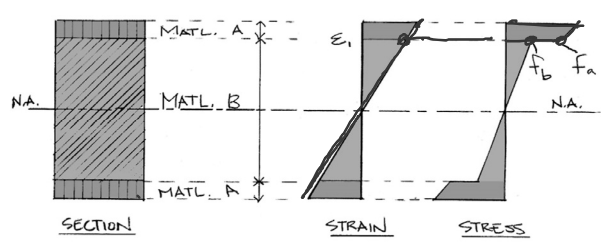
Therefore, the strains will be the same in each material under **axial load**.




Axial

In **flexure** the strains are the same as in a homogeneous section, i.e. linear.

In flexure, if the two materials are at the same distance from the N.A., they will have the same strain at that point because both materials share the same strain diagram. We say the strains are "compatible".



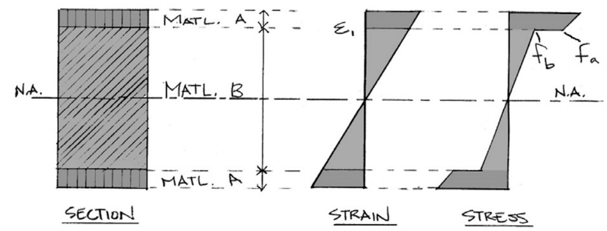

Flexure

**Stress** = E x Strain
So stress will be higher if E is higher.

# Strain Compatibility (cont.)

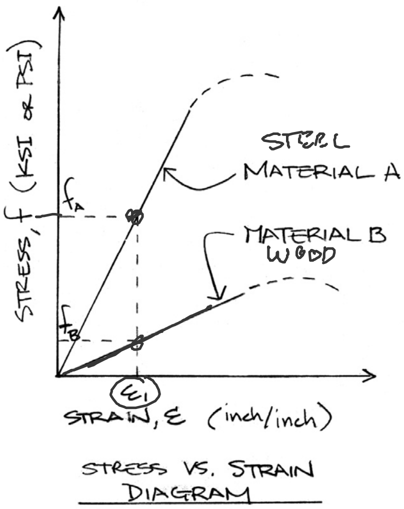
The stress in each material is determined by using Young's Modulus

$$\sigma = E\varepsilon$$




flexure

Care must be taken that the elastic limit of each material is not exceeded. The elastic limit can be expressed in either stress or strain.

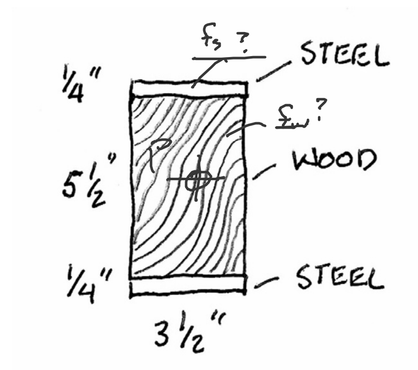





# Axial Compression

## Stress Analysis Procedure

Determine the safety (pass or fail) of the composite short pier under an axial load.



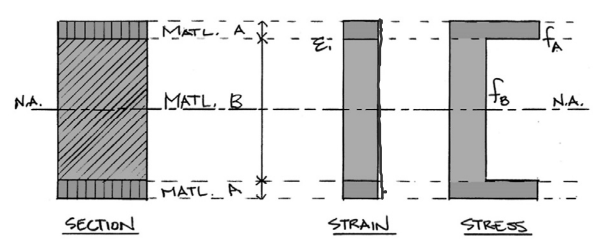

By Considère's Law:

1. P = f A = fs As + fw Aw
   (the actual stresses are unknown)

2. ∈pier = ∈s = ∈w (equal strains)
   ∈ = f / E
   fs / Es = fw / Ew
   fs = (Es / Ew) fw

3. Substitute [(Es / Ew) fw] for fs into the original equation and solve for fw

4. Use second equation to solve fs



## Example - Axial Compression

Pass/Fail Analysis:

Given: Load = 50 kips

Section: Aw = 19.25 in$^2$

Ew = 1000 ksi

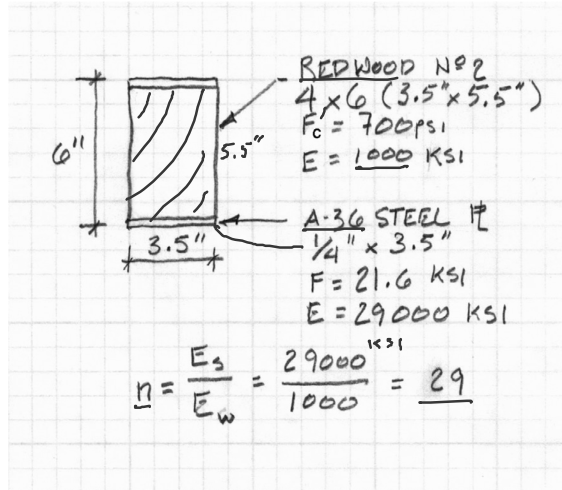
As = 2x 0.875 = 1.75 in$^2$

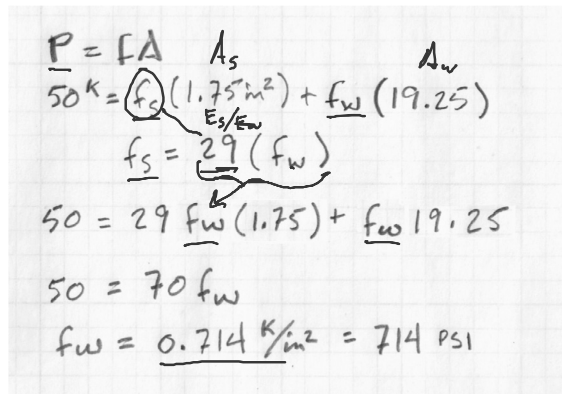
Es = 29000 ksi

Braced against buckling.

Req'rd: actual stress in the material

1. P = f A = fs As + fw Aw
   (the actual stresses are unknown )
2. fs = (Es / Ew) fw
3. P = (Es / Ew) fw As + fw Aw
4. Solve for fw



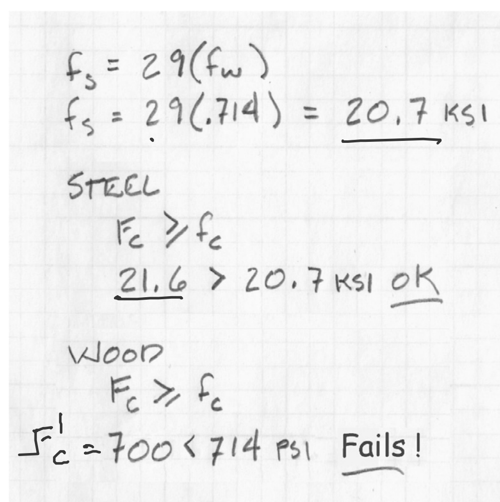



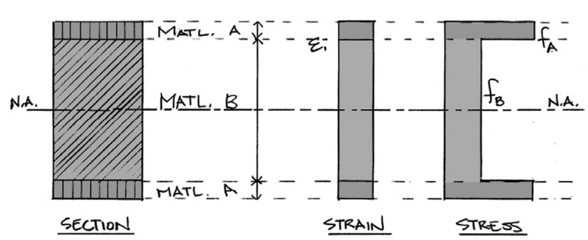



## Example - Axial Compression

Pass/Fail Analysis:

5. Use second equation to solve for fs

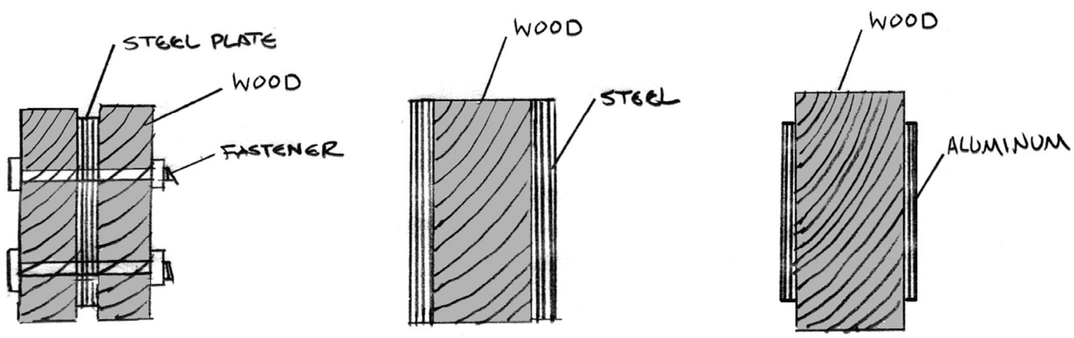


# Flitched Beams & Scab Plates

**Advantages**

- Compatible with the wood structure,
  i.e. can be nailed
- Easy to retrofit to existing structure
- Lighter weight than a steel section
- Stronger than wood alone
  - Less deep than wood alone
  - Allow longer spans
- The section can vary over the length of the span to optimize the member (e.g. scab plates)
- The wood stabilizes the thin steel plate

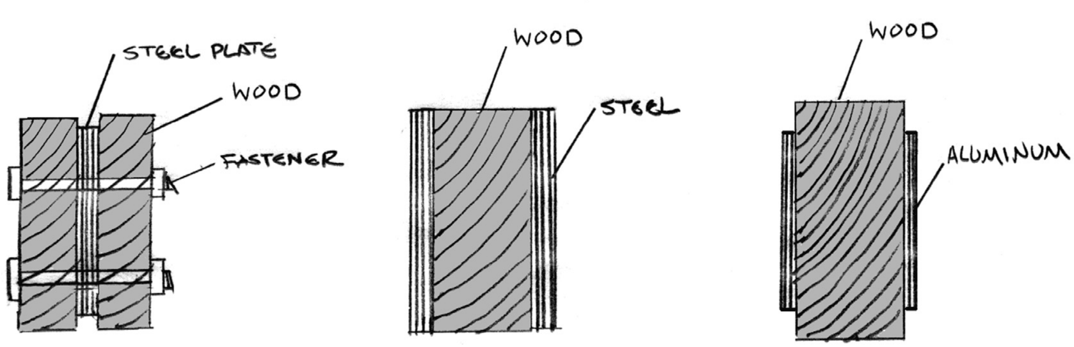





# Flitched Beams & Scab Plates

**Disadvantages**

- More labor to make – expense. Flitched beams requires shop fabrication or field bolting.
- Often replaced by Composite Lumber which is simply cut to length – less labor
  - Glulam
  - LVL
  - PSL
- Flitched Beams are generally heavier than Composite Lumber



# Flexure Stress using Transformed Sections

In the basic flexural stress equation, I is derived based on a homogeneous section. Therefore, to use the stress equation one needs to “transform” the composite section into a homogeneous section.

$$f_b = \frac{Mc}{I}$$

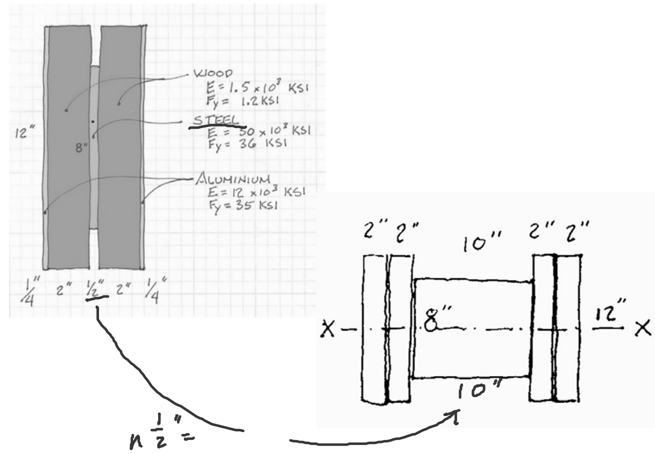
Homogeneous Section

$$f_b = \frac{Mc\ n}{I_{tr}}$$

Transformed Section

For the new “transformed section” to behave like the actual section, the stiffness of both would need to be the same.

Since Young’s Modulus, E, represents the material stiffness, when transforming one material into another, the area of the transformed material must be scaled by the ratio of one E to the other.




The scale factor is called the modular ratio, n.

$$n = \frac{E_A}{E_B}$$

Transformed material STEEL

Base material WOOD

In order to also get the correct stiffness for the moment of Inertia, I, only the width of the geometry is scaled. Using I from the transformed section ($I_{TR}$) will then give the same flexural stiffness as in the original section.



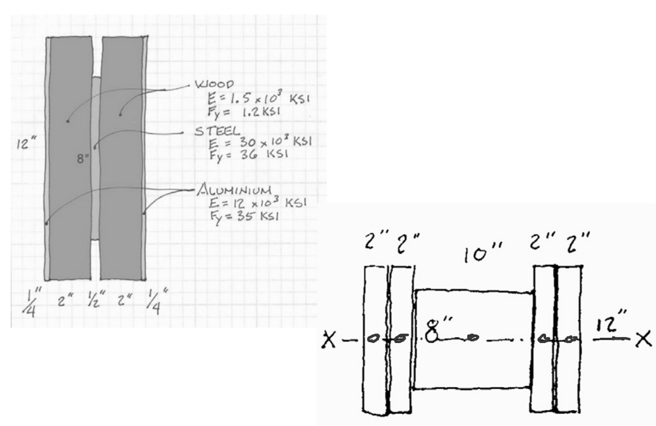
# Calculate the Transformed Section, $I_{TR}$

1. Use the ratio of the E modulus from each material to calculate a modular ratio, n.
2. Usually the softer (lower E) material is used as a base (denominator). Each material combination has a different n.
3. Construct a transformed section by scaling the width of each material by its modular, n.
4. $I_{tr}$ is calculated about the N.A.
5. If needed, separate transformed sections must be created for each axis (x-x and y-y)

Transformed material

$$n_A = \frac{E_A}{E_B}, \text{ and } \quad n_C = \frac{E_C}{E_B} \qquad n_B = \frac{E_B}{E_B}$$

Base material




$$I_{tr} = \sum I + \sum Ad^2$$

# Flitched Beam Analysis Procedure

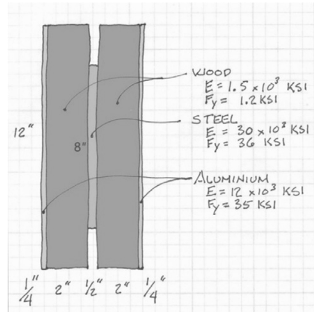
1. Determine the modular ratio(s). Usually the softer (lower E) material is used as a base (denominator). Each material has a different n.

Transformed material

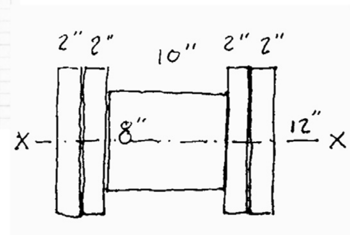
$$n_A = \frac{E_A}{E_B} \text{ and } n_C = \frac{E_C}{E_B}$$

Base material

2. Construct a transformed section by scaling the width of the material by its modular, n.




3. Determine the Centroid and Moment of Inertia of the transformed section.




4. Calculate the flexural stress in **each** material separately using:

$$f_b = \frac{Mc\ n}{I_{tr}}$$

$$I_{tr} = \sum I + \sum Ad^2$$

Transformation equation or solid-void



# Analysis Example – pass / fail

For the composite section, find the maximum flexural stress level in each laminate material.

$$f_b = \frac{Mc\ n}{I_{tr}}$$

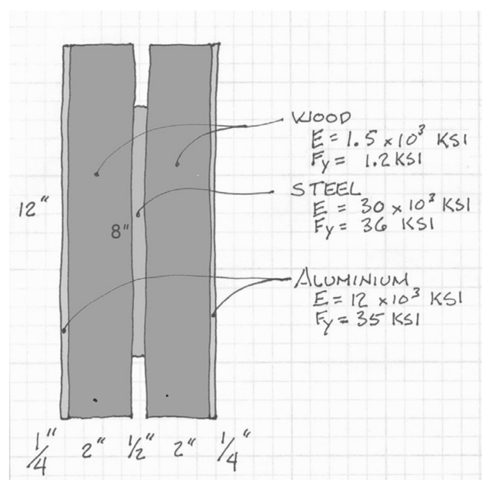
1. Determine the modular ratios for each material.

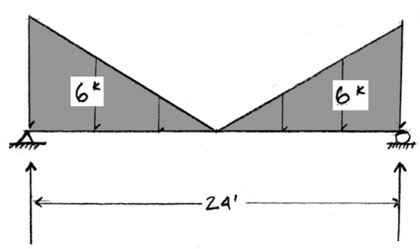
Use wood (the lowest E) as base material.

$$n_{WOOD} = \frac{1.5}{1.5} = 1.0$$

$$n_{AL} = \frac{12}{1.5} = 8.0$$

$$n_{ST} = \frac{30.}{1.5} = 20.$$



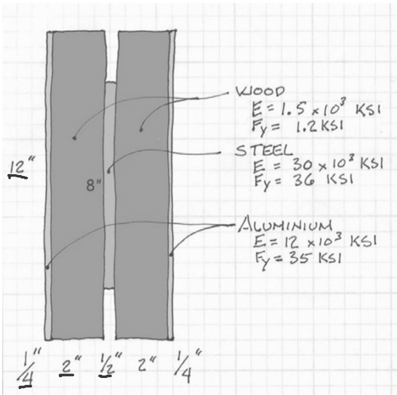




LOADING DIAGRAM

# Analysis Example cont.:

2. Construct a transformed section.

   Determine the transformed width of each material.




$$n_{WOOD} = \frac{1.5}{1.5} = 1.0$$

$$n_{AL} = \frac{12}{1.5} = 8.0$$

$$n_{ST} = \frac{30}{1.5} = 20.$$

ALUM.

$$t = 1/4''$$

$$t_{tr} = 1/4 \times n_{AL} = 1/4\,(8.0) = 2.0''$$

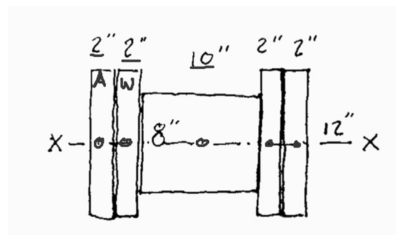
STEEL

$$t = 1/2''$$

$$t_{tr} = 1/2 \times n_{ST} = 1/2\,(20) = 10.''$$

WOOD

$$t = t_{tr} = 2''$$

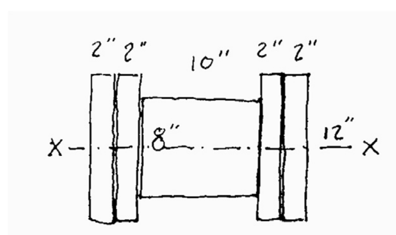



Transformed Section



# Analysis Example cont.:

2. Construct a transformed section.

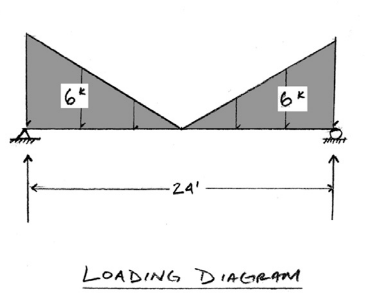


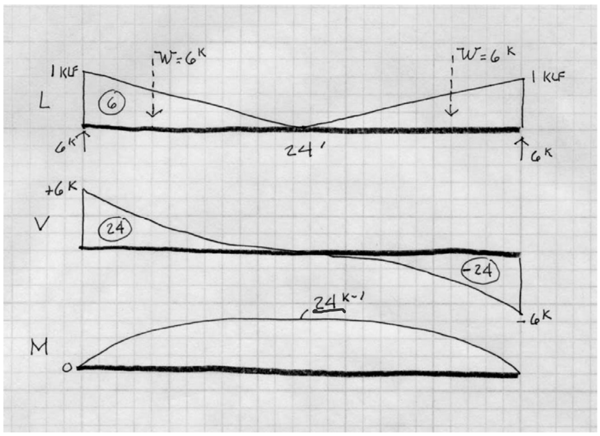

3. Calculate the Centroid and the Moment of Inertia for the transformed section.

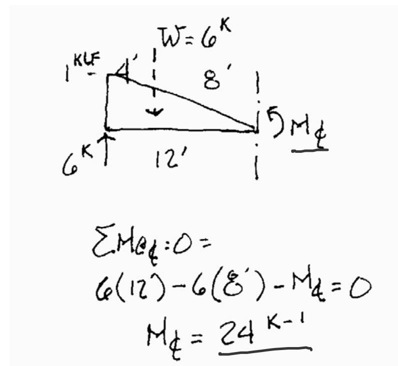
$$I_{tr} = \frac{8\ 12^3}{12} + \frac{10\ 8^3}{12} = 1152 + 426 = 1578\ in^4$$

# Analysis Example cont.:

Find the maximum moment.




by diagrams or by summing moments



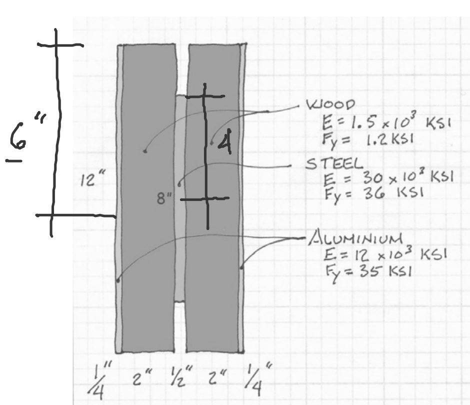
# Analysis Example cont.:

Actual Stresses:

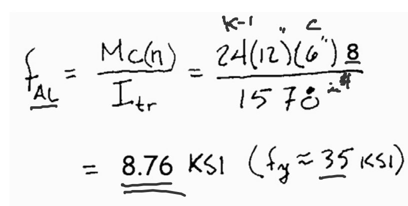
4. Calculate the stress for each material using stress equation with the transformed moment of inertia.

$$f_b = \frac{\mathrm{M}c\ n}{\mathrm{I}_{\mathrm{tr}}}$$

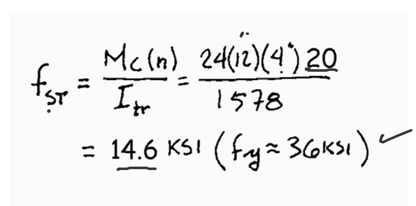
Compare the stress in each material to limits of yield stress or the safe allowable stress.



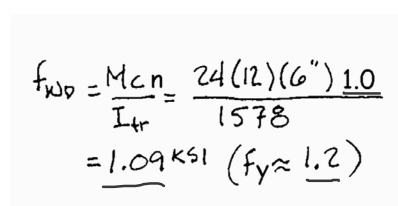

n = 12/1.5 = 8




n = 30/1.5 = 20



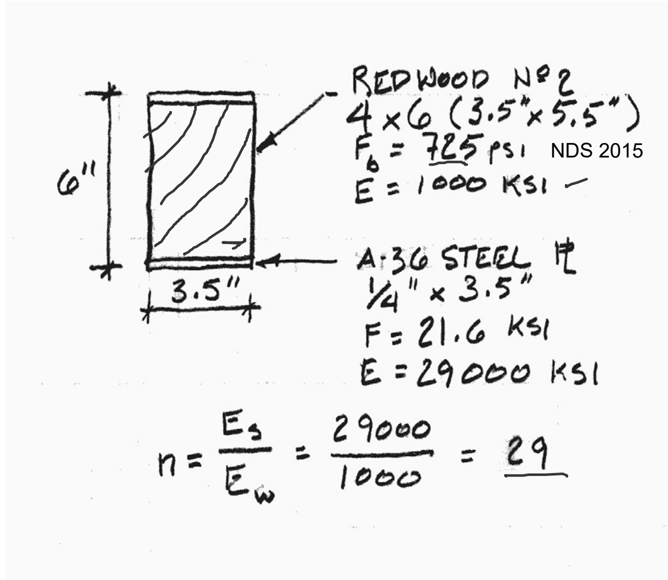

n = 1.5/1.5 = 1



# Capacity Analysis (ASD) Flexure

Given
- Dimensions
- Material

Required
- Load capacity




1. Determine the modular ratio. It is usually more convenient to transform the stiffer material.



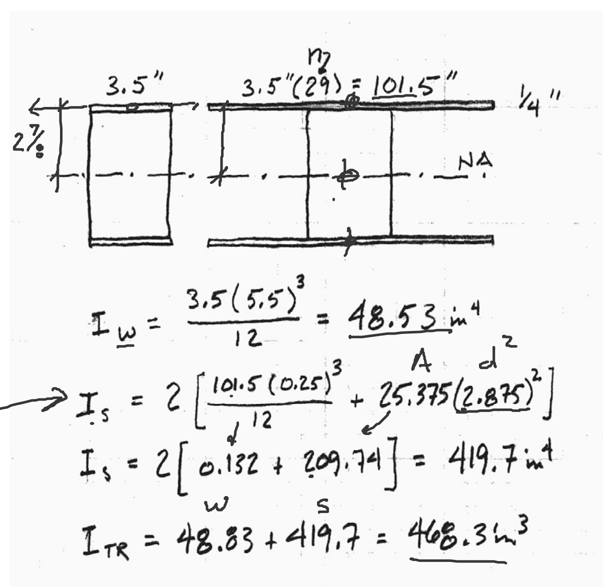
# Capacity Analysis (cont.)

2. Construct the transformed section. Multiply all widths of the transformed material by n. The depths remain unchanged.

3. Calculate the transformed moment of inertia, $I_{tr}$.

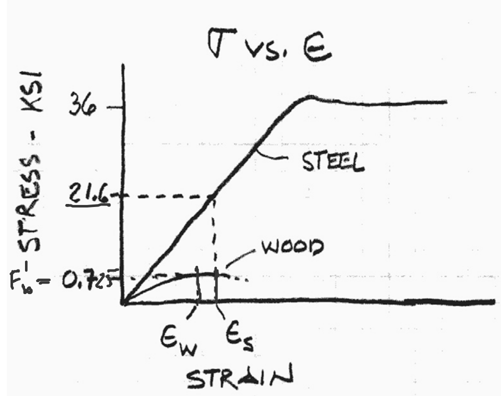
$$I_{tr} = \sum I + \sum Ad^2$$

25.375 + 8.265 = 33.64

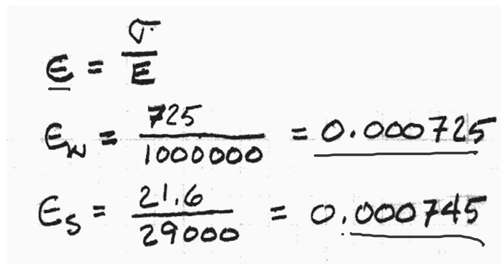


# Capacity Analysis (cont.)

4. Calculate the allowable strain based on the allowable stress for the material.




$$\varepsilon_{allow} = \frac{F_{allow}}{E}$$

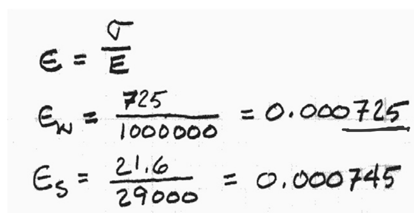


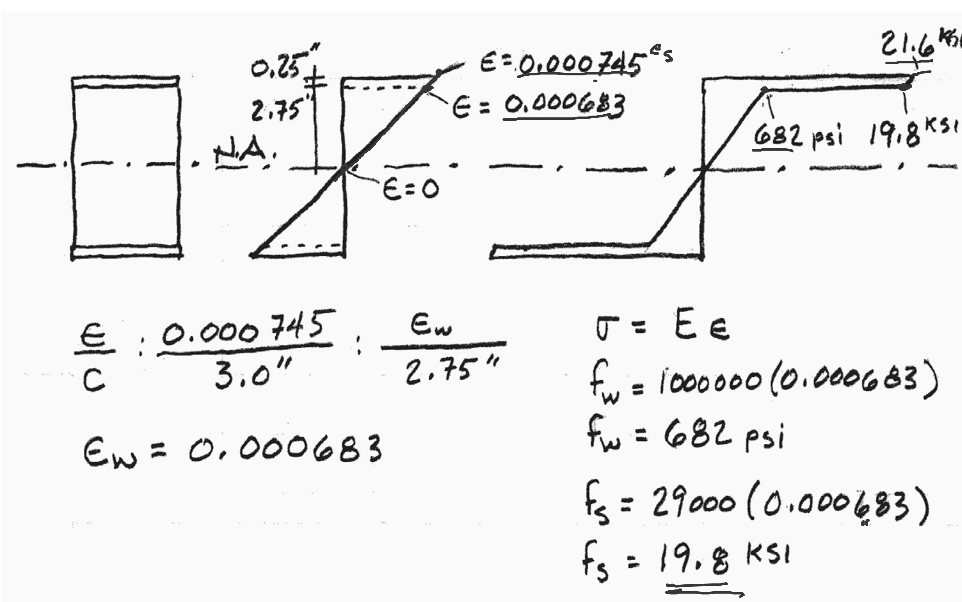



# Capacity Analysis (cont.)

5. Construct a strain diagram to find which of the two materials will reach its limit first. The diagram should be linear, and neither material may exceed its allowable limit.

Allowable Strains:



# Capacity Analysis (cont.)

6. The allowable moments (load capacity) may now be determined based on the stress of either material. Either stress should give the same moment if the strain diagram from step 5 is compatible with the stress diagram (they align and allowables are not exceeded).

$$M_s = \frac{f_s I_{TR}}{c\,n} = \frac{21.6\,(468.3)}{3\,(29)} = 116.2^{k\text{-}''}$$

$$M_w = \frac{f_w I_{TR}}{c} = \frac{0.682\,(468.3)}{2.75''} = 116.1^{k\text{-}''}$$

7. Alternatively, the controlling moment can be found without the strain investigation by using the maximum allowable stress for each material in the moment-stress equation. The **lower momen**t will be the first failure point and the controlling material.

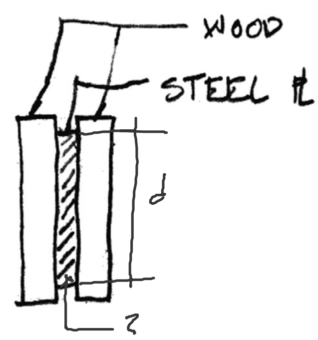
$$M_s = \frac{F_s I_{TR}}{c\,n} = \frac{21.6\,(468.3)}{3\,(29)} = 116.2^{k\text{-}''}$$

$$M_w = \frac{F_w I_{TR}}{c} = \frac{.725\,(468.3)}{2.75''} = 123.5^{k\text{-}''}$$

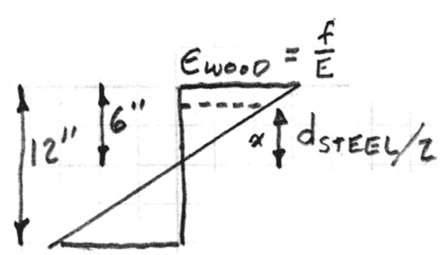


# Design Procedure:

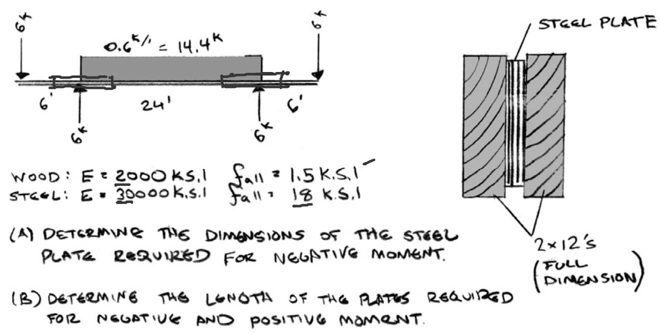
Given: Span and load conditions
Material properties
Wood dimensions
Req'd: Steel plate dimensions




1. Determine the required moment.
2. Find the moment capacity of the wood.
3. Determine the required capacity for steel.
4. Based on strain compatibility with wood, find the largest d for steel where $\epsilon_s < \epsilon_{allow}$.
5. Calculate the required section modulus for the steel plate. $S_x$.
6. Using d from step 4. calculate b (width of plate).
7. Choose final steel plate based on available sizes and check total capacity of the beam.



# Design Example:




1. Determine the required moment.
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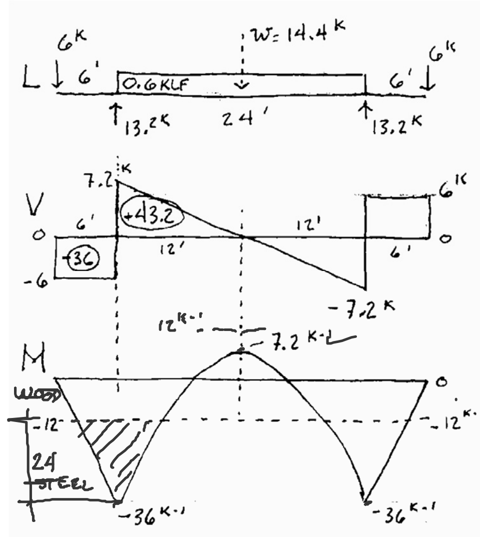
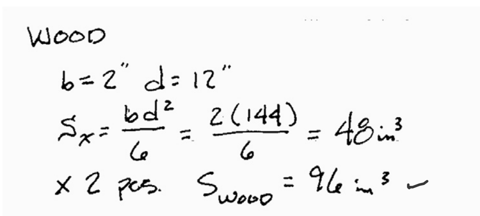
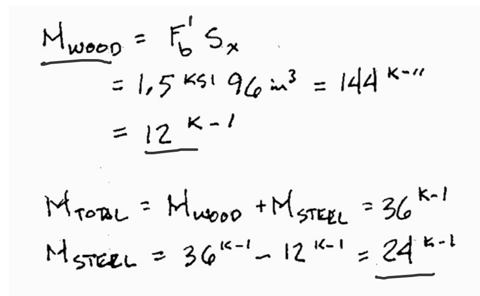


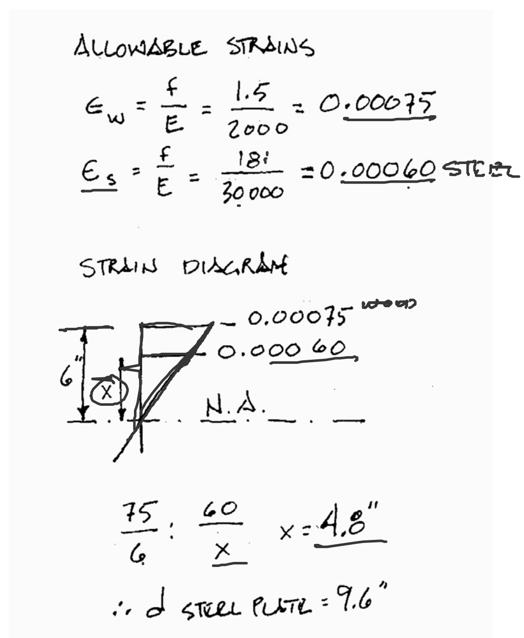





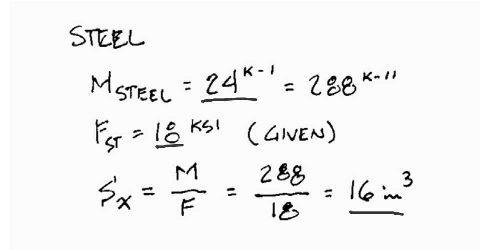

# Design Example cont:

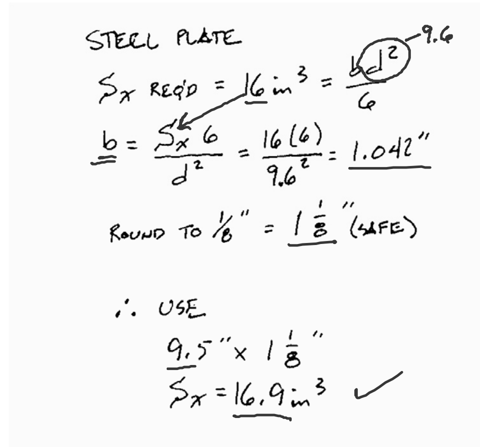
4. Based on strain compatibility with wood, find the largest d for steel where $\in_s \leq \in_{allowable.}$

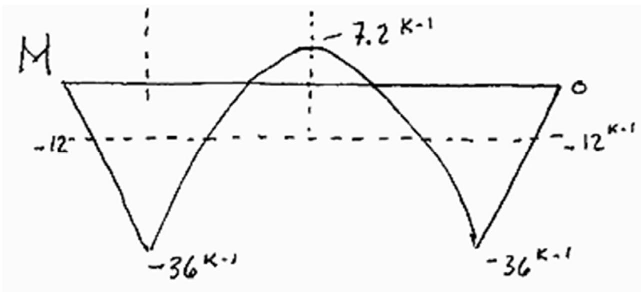


# Design Example cont:

5. Calculate the required section modulus for the steel plate.
6. Using d from step 4. calculate b (width of plate).
7. Choose final steel plate based on available sizes and check total capacity of the beam.



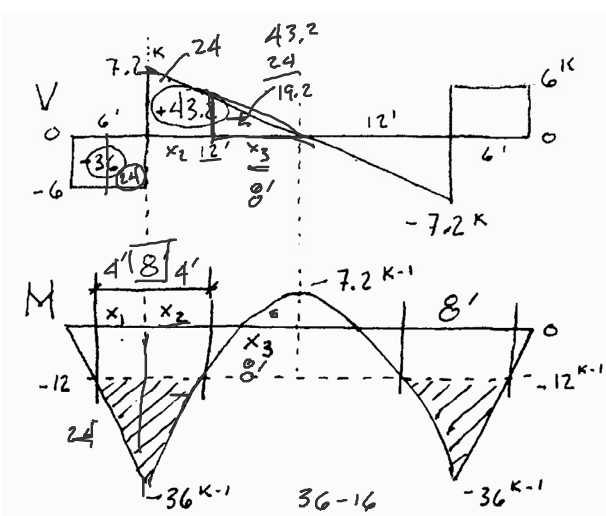



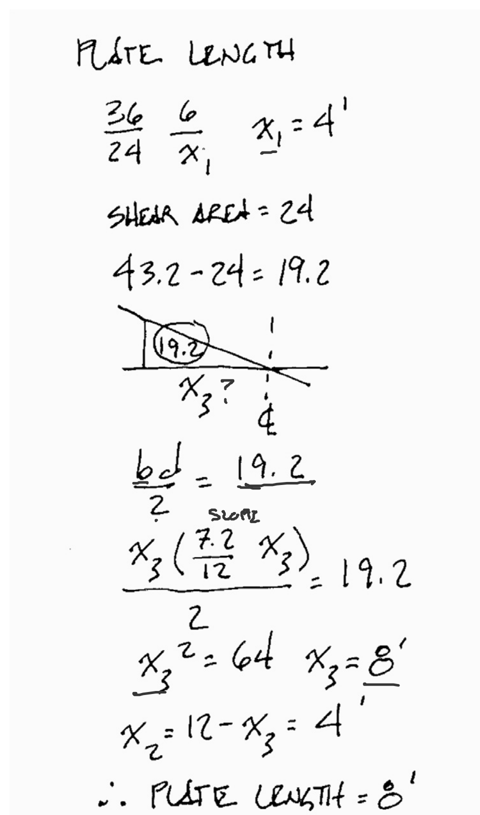






# Design Example cont:

8. Determine required length and location of plate.



# Applications:

## Renovation in Edina, Minnesota

Four 2x8 LVLs, with two 1/2" steel plates. 18 FT span
Original house from 1949
Renovation in 2006
Engineer: Paul Voigt

© Todd Buelow used with permission



# Applications:

## Renovation

Chris Withers House, Reading, UK 2007
Architect: Chris Owens, Owens Galliver
Engineer: Allan Barnes

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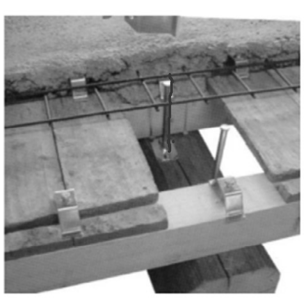
# Steel Sandwiched Beams

Also based on strain compatibility



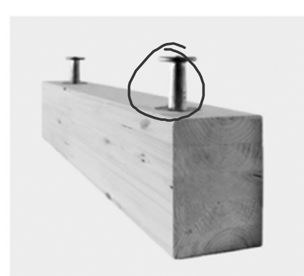
# Wood – Concrete Composites

For Slabs or Slabs + Beams



(a)

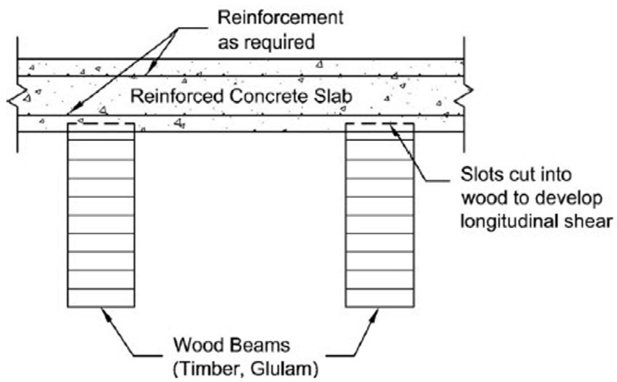
(b)



(c)

# Wood – Concrete Composites

For Slabs or Slabs + Beams



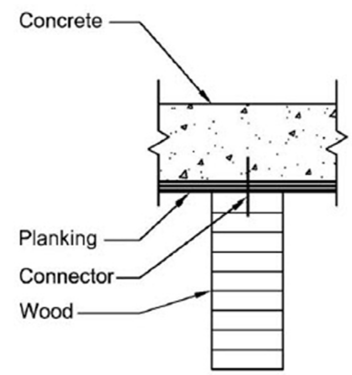




**Fig. 1.** Traditional wood–concrete composite T-beam deck (adapted from Clouston and Schreyer 2006)



# Wood – Concrete Composites

Comparison of connectors

from: Yeoh, David, et al., State of the Art on Timber-Concrete Composite Structures: Literature Review, *Journal fo Structureal Engineering*, ASCE Oct 2011

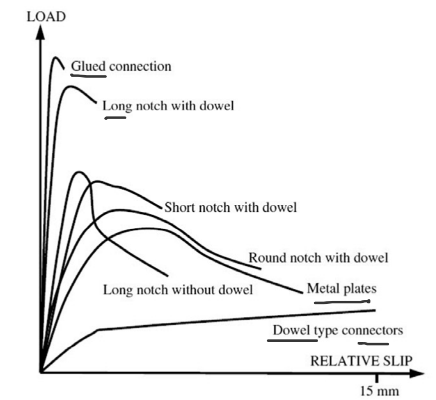



**Fig. 1.** Comparisons of different categories of connection systems

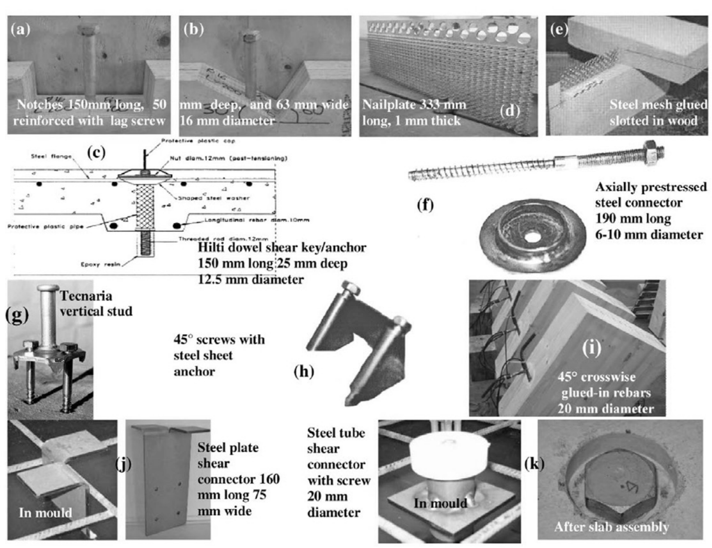



**Fig. 2.** Different connection systems: (a), (b), and (d) reprinted from Yeoh (2010); (c) reprinted from Gutkowski et al. (2004, ASCE); (e) reprinted from Clouston et al. (2005, ASCE); (f) Fig. 4: Axial prestressed steel connector, reprinted from Capozucca (1998) with permission from RILEM; (g) Fig. 1: "Tecnaria" connector, reprinted from Fragiacomo et al. (2007a) with permission from RILEM; (h) reprinted from Steinberg et al. (2003, ASCE); (i) reprinted from Kuhlmann and Aldi (2008) with permission from author; (j) and (k) Fig. 9: Steel plate shear connector SP+N and Fig. 7: Steel tube shear connector SST+S, reprinted from Lukaszewska et al. (2008) with permission from RILEM