

NDS 3.9.1 Bending and Axial Tension

two conditions

#### **3.9.1 Bending and Axial Tension**

Members subjected to a combination of bending and axial tension (see Figure 3G) shall be so proportioned that:

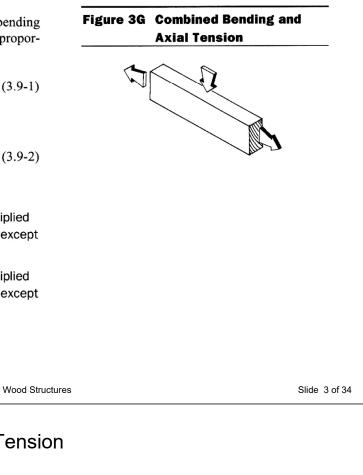
$$\frac{f_{t}}{F_{t}'} + \frac{f_{b}}{F_{b}} \le 1.0$$
 TENSION CRIT. (3.9-1)

and

$$\frac{f_{b} - f_{t}}{F_{b}^{**}} \le 1.0 \qquad \text{FLEXURE CRIT.} \qquad (3.9-2)$$

where:

- $F_{b}^{*}$  = reference bending design value multiplied by all applicable adjustment factors except  $C_{L}$
- $F_{b}$  = reference bending design value multiplied by all applicable adjustment factors except  $C_v$



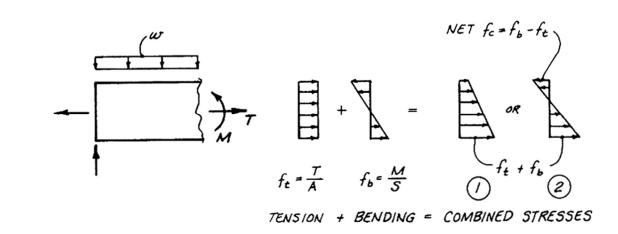
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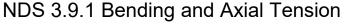
# NDS 3.9.1 Bending and Axial Tension NDS Equations

- CASE 1. Tension is critical. eq. 3.9-1 \* no  $C_L$
- CASE 2. Flexure is critical. eq. 3.9-2 \*\* no  $C_{\rm V}$

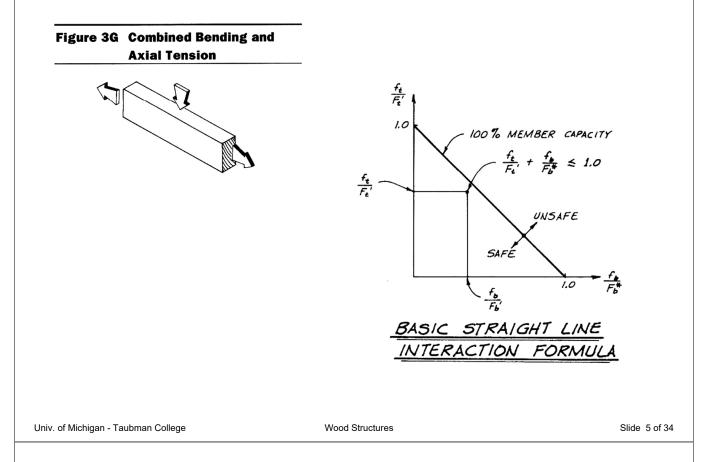
$$\frac{f_t}{F_{t'}} + \frac{f_b}{F_b^*} \le 1.0$$

$$\frac{f_b - f_t}{F_b * *} \le 1.0$$





tension + bending



# NDS 3.9.2 Bending and Axial Compression

two axis bending + compression

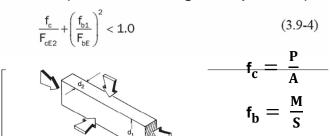
#### 3.9.2 Bending and Axial Compression

Members subjected to a combination of bending about one or both principal axes and axial compression (see Figure 3H) shall be so proportioned that:

$$\begin{bmatrix} \frac{f_{c}}{F_{c}'} \end{bmatrix}^{2} + \frac{f_{b1}}{F_{b1}' [1 - (f_{c}/F_{cE1})]} \\ + \frac{f_{b2}}{F_{b2}' [1 - (f_{c}/F_{cE2}) - (f_{b1}/F_{bE})^{2}]} \leq 1.0 \quad (3.9-3)$$

#### and

#### (Flatwise bending + compression)



where:

$$f_{c} < F_{cE1} = \frac{0.822 E_{min}'}{(\ell_{e1} / d_{1})^{2}}$$
 for either uniaxial edgewise bending or biaxial bending

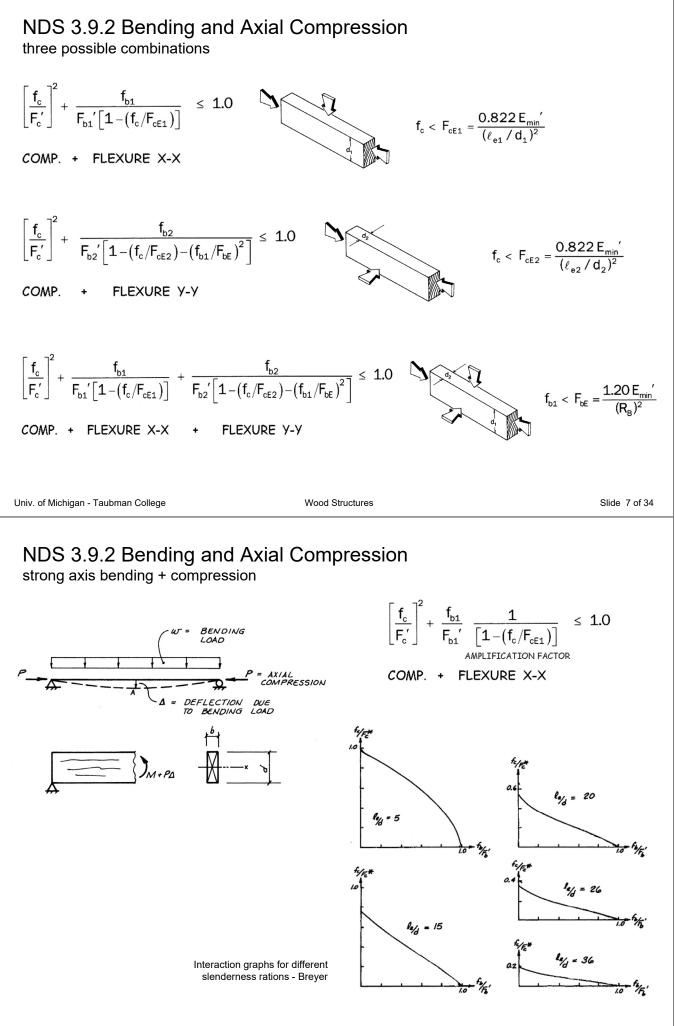
and

$$f_o < F_{oE2} = \frac{0.822 E_{min}'}{(\ell_{e2} / d_2)^2}$$
 for uniaxial flatwise  
bending or biaxial  
bending

and

$$f_{b1} < F_{bE} = \frac{1.20 E_{min}'}{(R_B)^2}$$
 for biaxial bending

- f<sub>b1</sub> = actual edgewise bending stress (bending load applied to narrow face of member) , psi
- $f_{b2}$  = actual flatwise bending stress (bending load applied to wide face of member) , psi
- $d_1$  = wide face dimension (see Figure 3H), in.
- d<sub>2</sub> = narrow face dimension (see Figure 3H), in.

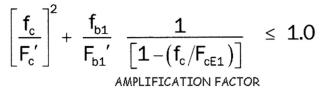


#### Second Order Stress "P Delta Effect"

With larger deflections this can become significant.

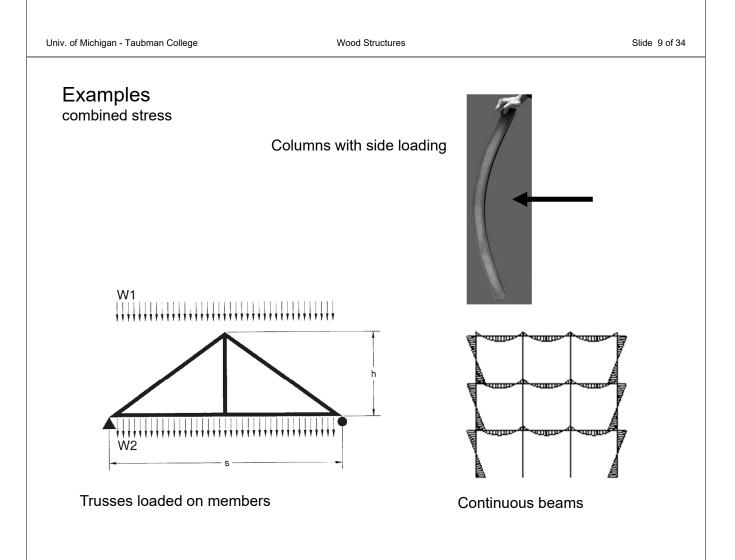
- 1. Eccentric load causes bending moment
- 2. Bending moment causes deflection,  $\Delta$
- 3.  $P \times \Delta$  causes additional moment

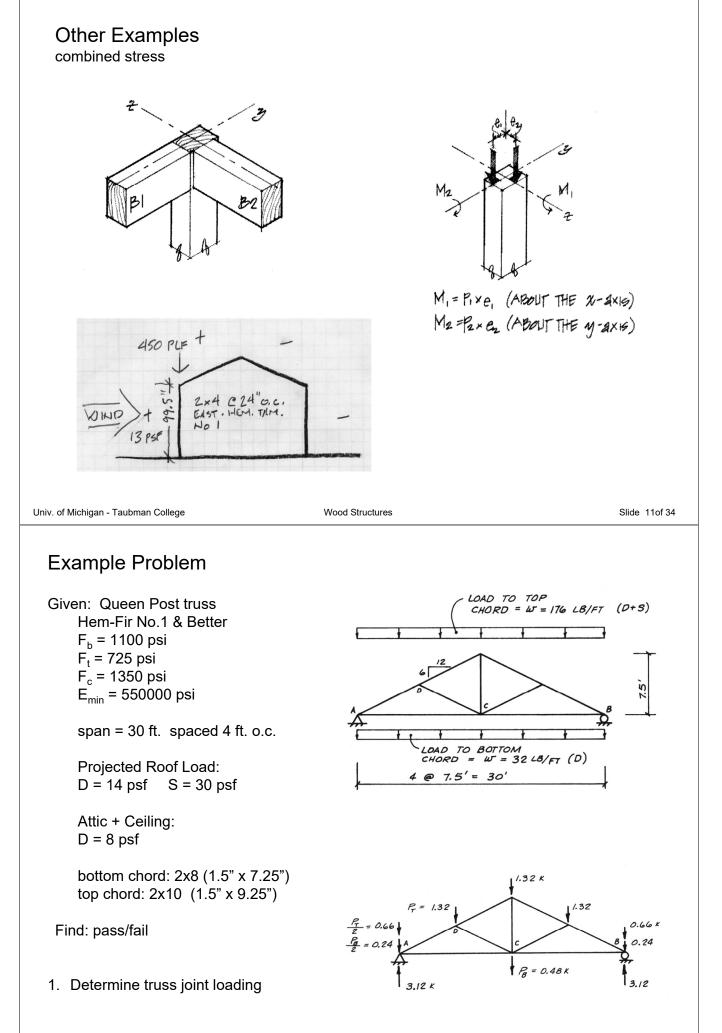
Accounted for by use of an amplification factor



COMP. + FLEXURE X-X







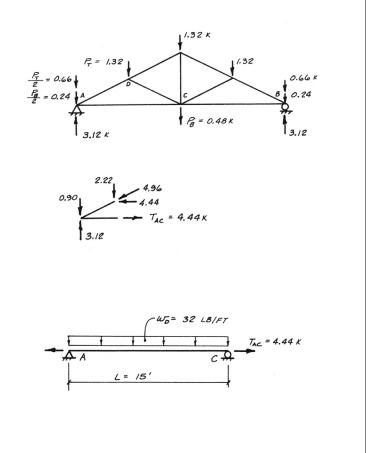
### Example

truss reactions and member forces

- Determine the external end reactions of the whole truss. The geometry and loads are symmetric, so each reaction is ½ of the total load.
- 3. Use an FBD of the reaction joint to find the **chord forces**. Sum the forces horizontal and vertical to find the components.

Top chord = 4.96 k compression

Bottom chord = 4.44 k tension



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D+S Load:

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D+S

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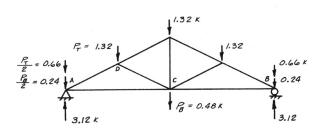
Example bottom chord 2x8 – bending + tension

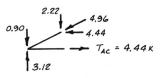
4. Determine controlling load case:

**Tension Force: D** 

 $P_t / C_D = 1.74 \text{ k} / 0.9 = 1.93$ 

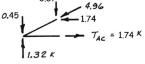
Tension Force: D+S  $P_t / C_D = 4.44 \text{ lbs} / 1.15 = 3.86 \text{ (controls)}$ 

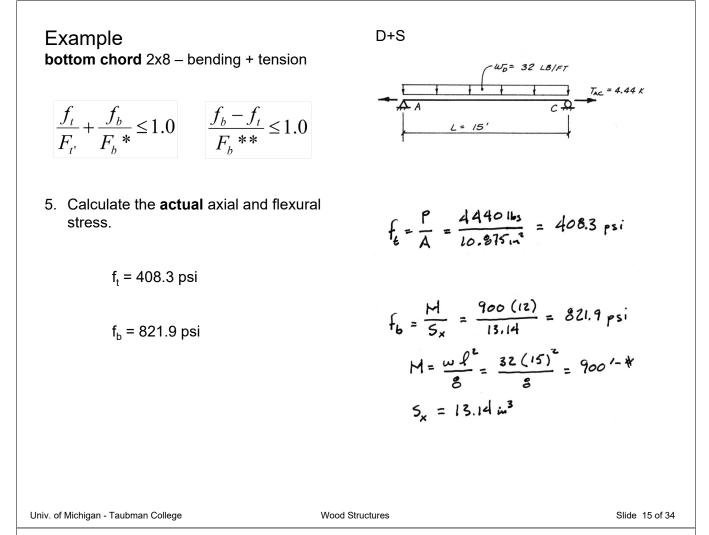




D alone

 $\frac{P_{T}}{P_{Z}} = 0.21$   $\frac{P_{T}}{P_{Z}} = 0.24$   $\frac{P_{T}}{P_{Z}} = 0.24$   $\frac{P_{T}}{P_{Z}} = 0.24$   $\frac{P_{T}}{P_{Z}} = 0.24$   $\frac{P_{T}}{P_{Z}} = 0.48 \times 0.24$   $\frac{P_{T}}{P_{Z}} = 0.48 \times 0.24$   $\frac{P_{T}}{P_{Z}} = 0.48 \times 0.24$ 





# Example bottom chord 2x8 – bending + tension

Hem-Fir No.1 & Better  $F_b = 1100 \text{ psi}$  $F_t = 725 \text{ psi}$ 

6. Determine **allowable** stresses using applicable factors:

Tension: D+S  $F_t' = F_t (C_D C_F)$  $F_t' = 725 (1.15 1.2) = 1000 \text{ psi}$ 

$$F_t$$
 = 1000 > 408.3 =  $f_t$  ok

Flexure: D+S  $F_{b}' = F_{b} (C_{D} C_{L} C_{F})$  $F_{b}' = 1100 (1.15 1.0 1.2) = 1518 psi$ 

$$F_{b}$$
' = 1518 > 821.9 =  $f_{b}$  ok

CL 15 1.0 BY 4.4.1 d/b = d, ENDS ARE HELD

Size Factors, C<sub>F</sub> F<sub>b</sub> Ft Fc Thickness (breadth) 2" & 3" 4" Grades Width (depth 2", 3", & 4" 1.5 1.5 1.5 1.15 Select 5" 1.4 1.4 1.4 1.1 Structural, 6" 1.3 1.3 1.3 1.1 8" 1.05 No.1 & Btr, 1.2 1.3 1.2 No.1, No.2, 10" 1.1 1.2 1.0 1.1 No.3 12" 1.0 1.1 1.0 1.0 0.9 14" & wider 1.0 0.9 0.9 2", 3", & 4" 1.1 1.1 1.1 1.05 Stud 5" & 6" 1.0 1.0 1.0 1.0 8" & wider Use No.3 Construction 2", 3", & 4" 1.0 1.0 1.0 1.0 Standard Utility 4" 1.0 1.0 1.0 1.0 0.4 2" & 3" 0.4 0.6

# Example bottom chord 2x8 – combined stress

#### **3.9.1 Bending and Axial Tension**

Members subjected to a combination of bending and axial tension (see Figure 3G) shall be so proportioned that:

$$\frac{f_{t}}{F_{t}} + \frac{f_{b}}{F_{b}} \le 1.0$$
 TENSION CRIT. (3.9-1)

and

$$\frac{f_{b}-f_{t}}{F_{b}^{**}} \leq 1.0 \qquad \text{FLEXURE CRIT.} \qquad (3.9-2)$$

where:

- $F_{b}^{*}$  = reference bending design value multiplied by all applicable adjustment factors except  $C_{L}$
- $F_{b}$  = reference bending design value multiplied by all applicable adjustment factors except  $C_v$

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{**}-**,

(2.9-1)

(3.9-2)

 $\frac{408.3}{1000} + \frac{321.9}{1518}$ 

0.4083 + 0.5414 = 0.95

0.95 × 1.0 pass

 $\frac{821.9 - 408.3}{1518} = 0.2724$ 

0.275 1.0 V PASS

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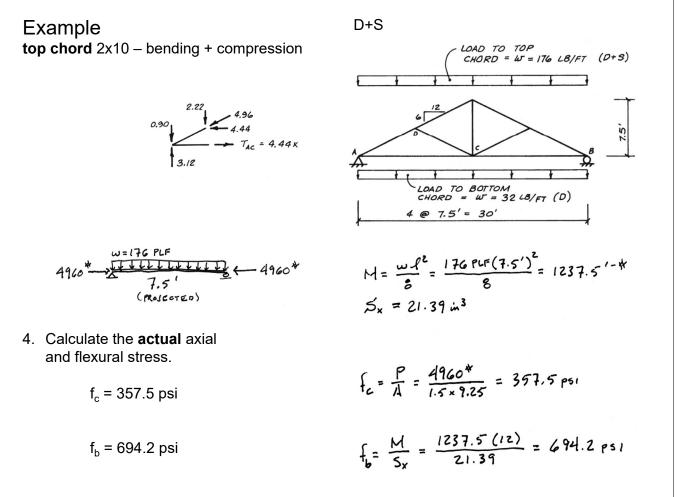
NET fc=fb-

 $\cap$ 

TENSION + BENDING = COMBINED STRESSES

2

 $f_b = \frac{M}{S}$ 



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#### Example top chord 2x10 – bending + compression

Hem-Fir No.1 & Better  $F_b = 1100 \text{ psi}$   $F_c = 1350 \text{ psi}$  $E_{min} = 550000 \text{ psi}$ 

5. Determine **allowable** stresses using applicable factors:

(compression: D+S)  $F_c' = F_c (C_D C_F C_P)$  $F_c' = 1350 (1.15 1.0 0.897) = 1392.7 \text{ psi} > 357.5$ 

(**flexure**: D+S)  
$$F_b' = F_b (C_D C_L C_F)$$
  
 $F_b' = 1100 (1.15 1.0 1.1) = 1392 psi > 694.2$ 

strong axis buckling

$$C_{P}$$

$$l_{e} = 8.385' \quad d = 9.25''$$

$$l_{e/d} = \frac{8.385(12)}{9.25} = 10.88$$

$$F_{cE} = \frac{0.822 \text{ Emin}}{(l_{e/d})^{2}} = \frac{0.822(55000)}{10.88^{2}} = 3820 \text{ psi}$$

$$F_{c}^{*} = 1350(1.15 1.0) = 1552.5 \text{ psi}$$

$$F_{c}^{*} = \frac{3820}{1552} = 2.46 \quad c = 0.8$$

		F <sub>b</sub>		Ft	Fc
		Thickness (breadth)			
Grades	Width (depth)	2" & 3"	4"		
	2", 3", & 4"	1.5	1.5	1.5	1.15
Select	5"	1.4	1.4	1.4	1.1
Structural,	6"	1.3	1.3	1.3	1.1
No.1 & Btr,	8"	1.2	1.3	1.2	1.05
No.1, No.2,	10"	1.1	1.2	1.1	1.0
No.3	12"	1.0	1.1	1.0	1.0

 $C_L$ 

The top chord is braced by the plywood sheathing so  $C_1 = 1.0$ 

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 $F_{CE_1} = \frac{0.822(550000)}{(100.6''/9.25'')}$ 

= 3820.7 PSI

Eq. 3.9-3

1.0

Example

top chord 2x10

$$\left[\frac{f_{c}}{F_{c}'}\right]^{2} + \frac{f_{b1}}{F_{b1}' \left[1 - (f_{c}/F_{cE1})\right]} \leq$$

COMP. + FLEXURE X-X

where:

$$f_{c} < F_{cE1} = \frac{0.822 E_{min}'}{(\ell_{e1} / d_{1})^{2}}$$
  
EULER 1  
for either uniaxial edge-  
wise bending or biaxial  
bending

and

$$f_{c} < F_{cE2} = \frac{0.822 E_{min}}{(\ell_{e2} / d_{2})^{2}} \qquad \begin{array}{l} \text{EULER 2} \\ \text{for uniaxial flatwise} \\ \text{bending or biaxial bend-ing} \end{array}$$

and

$$f_{\text{b1}} < \; F_{\text{bE}} = \frac{1.20 \, {E_{\text{min}}}'}{(R_{\text{B}})^2}$$

 $f_{b1}$  = actual edgewise bending stress (bending load applied to narrow face of member)

LTB

- $f_{b2}$  = actual flatwise bending stress (bending load applied to wide face of member)
- $d_1$  = wide face dimension (see Figure 3H)
- $d_2 = narrow face dimension (see Figure 3H)$

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$$\frac{\left[\frac{f_{c}}{F_{c}}\right]^{2}}{\left[\frac{f_{c}}{F_{c}}\right]^{2}} = \frac{\left[\frac{357.5}{1392.7}\right]^{2}}{\left[\frac{1392.7}{1392.7}\right]} = 0.0659$$

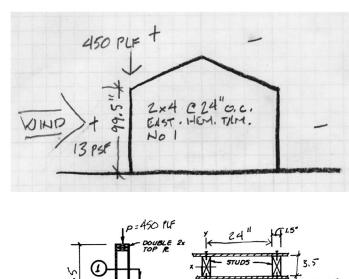
$$\frac{f_{61}}{F_{61}} = \frac{694.2}{1392} = 0.4987$$

$$\frac{1}{1 - (357.5/3820)} = \frac{1}{0.906}$$
  
0.4987 (1.103) = 0.550

Stud wall example

Exterior stud wall under bending + axial compression

- 1. Determine load per stud
- 2. Use axial load and moment to find actual stresses fc and fb
- 3. Determine load factors
- 4. Calculate factored stresses
- 5. Check NDS equation 3.9-3

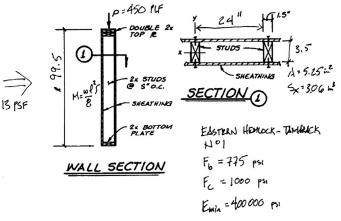


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# Combined Stress in NDS stud wall

Exterior stud wall under bending + axial compression



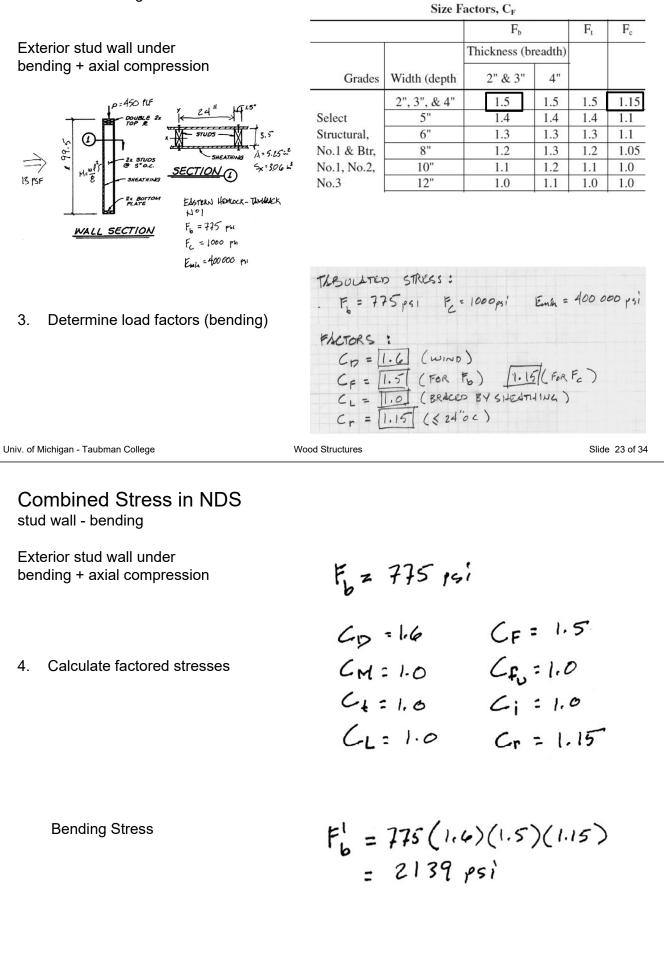
- 1. Determine load per stud
- 2. Use axial load and moment to find actual stresses  $\rm f_c$  and  $\rm f_b$

$$E_{\text{min}} = \frac{400000}{\text{Fil}} \text{ Fil}$$

$$P = \frac{1}{450} \frac{1}{12} = \frac{1}{450} \frac{1}{12} = \frac{1}{12} \frac{1}{12} = \frac{1}{12} \frac{1}{12} \frac{1}{12} = \frac{1}{12} \frac{1}{12} \frac{1}{12} = \frac{1}{12} \frac{1}{12} \frac{1}{12} \frac{1}{12} = \frac{1}{12} \frac{1}{$$

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stud wall - bending



stud wall - compression

Exterior stud wall under  
bending + axial compression  

$$C_{p} = \frac{1 + (F_{ct}/F_{c}^{*})}{2c} - \sqrt{\left[\frac{1 + (F_{ct}/F_{c}^{*})}{2c}\right]^{2}} - \frac{F_{ct}/F_{c}^{*}}{c}$$

$$\int_{q}^{p} = 900^{\frac{1}{2}} P + V^{\frac{1}{2}}$$

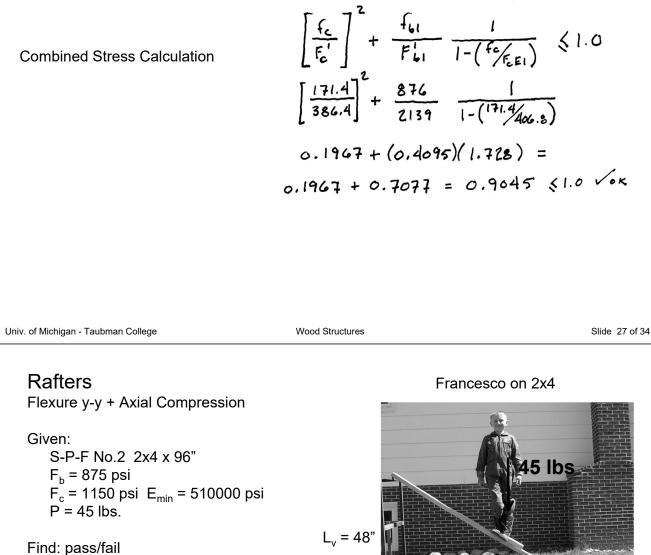
$$qq. 5''$$

$$(strong axis)$$

$$P^{\frac{1}{2}} = 1000(1.6 \times (1.6 \times (1.6$$

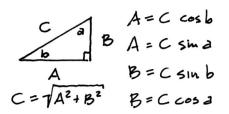
stud wall - combined stress

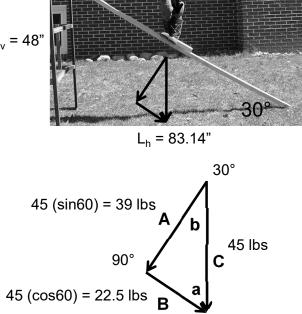
Exterior stud wall under bending + axial compression



Find normal and axial components of the load.

Axial = 22.5 lbs Normal = 39 lbs





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60°

## Rafters

Flexure y-y + Axial Compression

Given:

S-P-F No.2 2x4 x 96"  $F_b = 875 \text{ psi}$   $F_c = 1150 \text{ psi} \text{ E}_{min} = 510000 \text{ psi}$  P = 45 lbs. (as roof Lr) Find: pass/fail

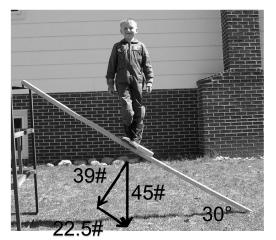
Axial = 22.5 lbs Normal = 39 lbs

Actual stress:

COMPRESSION:  

$$f_c = A = \frac{22.5}{5.25} = 4.285 \text{ psi}$$
  
 $A = 5.25 \text{ m}^2$   
 $P = 22.5$ 

Francesco on 2x4



FLEXURE Y-Y  $f_{bz} = \frac{M}{S'_{y}} = \frac{936}{1.315} = 712.5 \text{ psi}$   $M = \frac{PL}{4} = \frac{39(96')}{4} = 936''-*$  $S'_{y} = 1.313 \text{ m}^{3}$ 

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### Rafters

Flexure y-y + Axial Compression

S-P-F No.2 2x4 x 96"  $F_b$  = 875 psi F<sub>c</sub> = 1150 psi  $E_{min}$  = 510000 psi

Determine factored allowable stresses:

Compression

$$F_{c}^{I} = F_{c} (C_{p} C_{F} C_{P})$$

$$F_{c} = 1150 \text{ psi}$$

$$C_{p} (L_{r}) = 1.25$$

$$C_{F} = 1.15$$

$$F_{cE} = \frac{0.822 \text{ Emin}'}{(P_{e}/d)^{2}} = \frac{0.822 (510000)}{(96/1.5)^{2}} = 102.3 \text{ psi}$$

$$F_{c}^{*} = 1150 (1.25 1.15) = 1653 \text{ psi}$$

$$F_{cE}/F_{c}^{*} = \frac{102.3}{1653} = 0.06191$$

$$C_{p} = 0.0611$$

$$F_{c}^{i} = 1150 (1.25 1.15 0.0611) = 101.0 \text{ psi}$$

Flat Use Factor, C<sub>fu</sub>

Bending design values adjusted by size factors are based on edgewise use (load applied to narrow face). When dimension lumber is used flatwise (load applied to wide face), the bending design value,  $F_b$ , shall also be permitted to be multiplied by the following flat use factors:

Flat Use Factors, C <sub>fu</sub>					
Width	Thickness (breadth)				
(depth)	2" & 3"	4"			
2" & 3"	1.0	_			
4"	1.1	1.0			
5"	1.1	1.05			
6"	1.15	1.05			
8"	1.15	1.05			
10" & wider	1.2	1.1			

#### Flexure y-y

$$F_{b2}' = F_{b} (C_{p} C_{L} C_{F} C_{fu})$$
  

$$F_{b} = 875 \text{ poi}$$
  

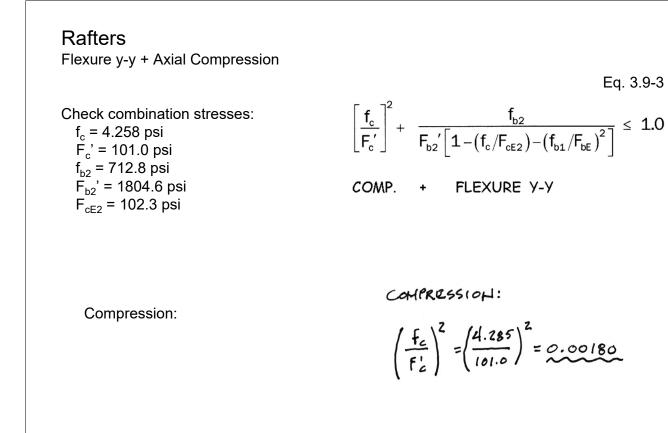
$$C_{p} (L_{F}) = 1.25$$
  

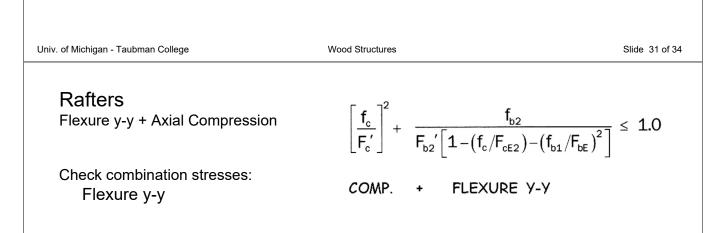
$$C_{F} = 1.5$$
  

$$C_{fu} = 1.1$$
  

$$C_{L} = 1.0$$
  

$$F_{b2}' = 875 (1.25 \ 1.0 \ 1.5 \ 1.1) = 1804.6 \text{ psi}$$





$$f_{c} = 4.285 \text{ psi}$$

$$f_{ce2} = 102.3 \text{ psi}$$

$$\frac{f_{c}}{F_{ce2}} = \frac{4.285}{102.3} = 0.04188$$

$$f_{b1} = \frac{M}{S_{x}} = \frac{0}{3.06} = 0$$

(in this example there is no strong axis bending, so the term is zero)

$$\frac{f_{b2}}{F_{b2}} = \frac{712.8}{1804.6} = 0.395$$

AMPLIFICATION FACTOR:

$$\frac{1}{1 - (f_{c}/F_{cE2}) - (f_{b1}/F_{bE})^{2}}$$

$$\frac{1}{1 - (\frac{4.235}{1.02.3}) - (\frac{0}{F_{bE}})^{2}} = \frac{1}{1 - 0.0415 - 0}$$

$$\frac{1}{0.9582} = 1.043$$

Rafters

Flexure y-y + Axial Compression

Check combination stresses:

Eq. 3.9-3

$$\left[\frac{f_{c}}{F_{c}'}\right]^{2} + \frac{f_{b2}}{F_{b2}' \left[1 - (f_{c}/F_{cE2}) - (f_{b1}/F_{bE})^{2}\right]} \leq 1.0$$

COMP. + FLEXURE Y-Y

CONPRESSION + FLEXURE Y-Y 0.0018 + 0.412 = 0.414 0.414 < 1.0 Vok

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#### Rafters

Flexure y-y + Axial Compression

Check combination stresses:

Eq. 3.9-4

