Architecture 544

Wood Structures

Graphic Statics

Analysis

Design

CITIC Financial Center, SOM
Shenzhen Bay Super Headquarter

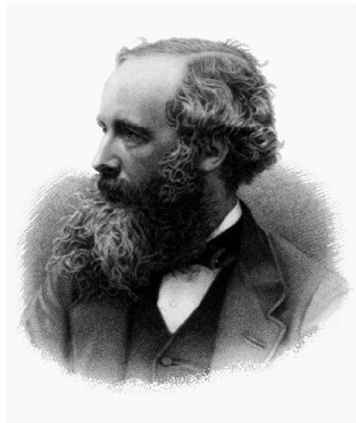# Development of Graphic Statics

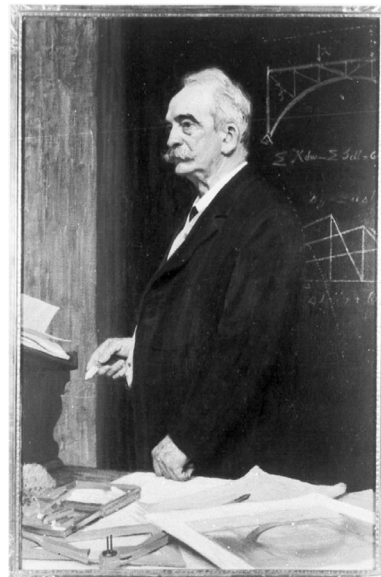James Clerk Maxwell 1869
M. Williot 1877
Otto Mohr 1887
Luigi Cremona 1890
Heinrich Müller-Breslau 1904

Otto Mohr

James Clerk Maxwell

Development of Graphic Statics

Wacław Zalewski & Edward Allen, 1998. *Shaping Structures: Statics*

Philippe Block, Matt DeJong and John Ochsendorf, 2006. *As Hangs the Flexible Line: Equilibrium of Masonry Arches.*

William Baker, 2015. *Combining Classic Theories with New Technologies*.

Caitlin Mueller, Corentin Fivet and John Ochsendorf, 2015. *Graphic Statics and Interactive Optimization for Engineering Education.*

William Baker   SOM

Caitlin Mueller   MIT

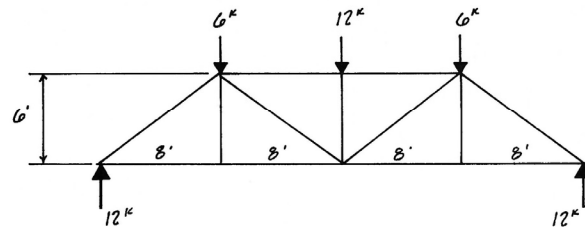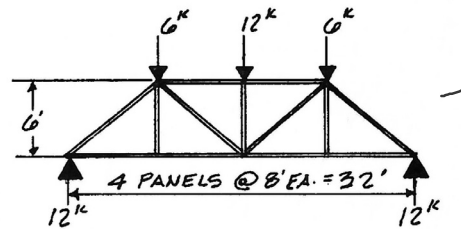John Ochsendorf   MIT

Philippe Block and Matt DeJong   ETH

Graphic Statics - Step 1
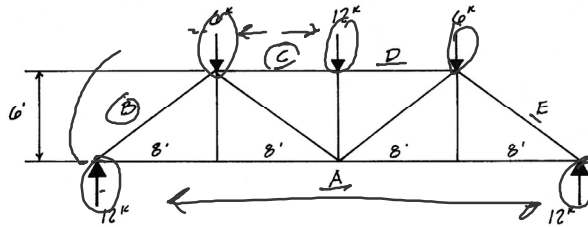
Draw the truss to scale

- Solve for all external forces on the truss.
- Make a simple line drawing of your truss.
- Draw it to scale.
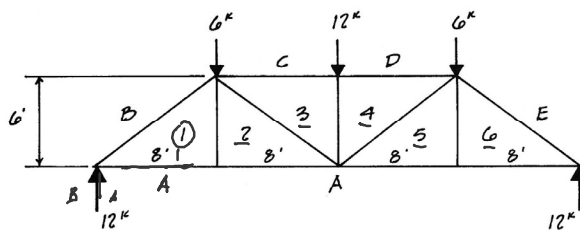
Graphic Statics - Step 2

Letter external spaces

- Label each space outside the truss with a letter.
- Start at bottom center (by convention).
- Continue in a clockwise direction around the outside of the truss.

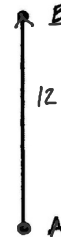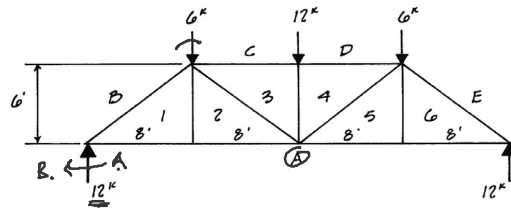# Graphic Statics - Step 3

Number internal spaces

- Number each space inside the truss.
- Number from left to right.

Graphic Statics - Step 4
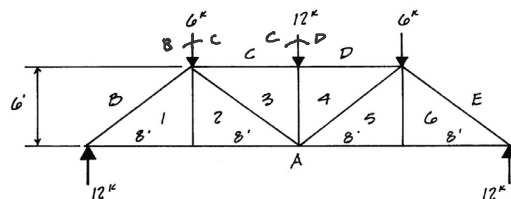
Draw the force diagram

- Start in the external space (A) and move clockwise to the next space (B).

- Note the direction and magnitude of the external force you cross (12k up).

- Draw this force to scale.

- The force starts with the tail (A) and continues to the head (B).

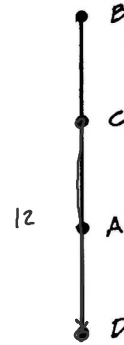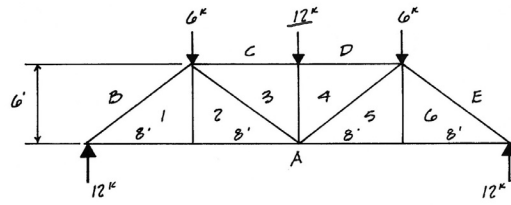# Graphic Statics - Step 4

Draw the force diagram

- Continue clockwise around the outside of the truss.

- Draw to scale the next force (6k downward) which is crossed as you move to the next space (C).

- Draw the tail at the first letter (B) and the head at the second (C).

- Draw the force in its actual direction and to scale.

Graphic Statics - Step 4

Draw the force diagram

- Continue clockwise around the outside of the truss.
- Draw to scale the next force (12k downward) which is crossed as you move to the next space (D).
- Draw the tail at the first letter (C) and the head at the second (D).
- Draw the force in its actual direction and to scale.

Graphic Statics - Step 4

Draw the force diagram

- Continue clockwise around the outside of the truss.
- Draw to scale the next force (6k downward) which is crossed as you move to the next space (E).
- Draw the tail at the first letter (D) and the head at the second (E).
- Draw the force in its actual direction and to scale.
- Draw the last force, 12k, starting at E and ending at A
- The head of the last force (A) should land on the tail of the first force (A). The external forces close to zero. If not, then there was some error in the forces (like the reactions may be wrong)
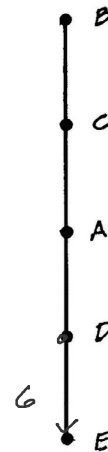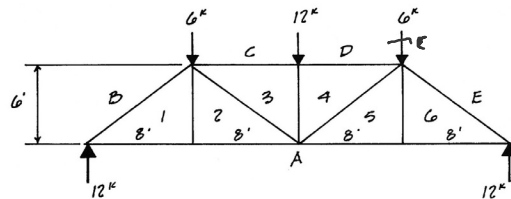
Graphic Statics - Step 5
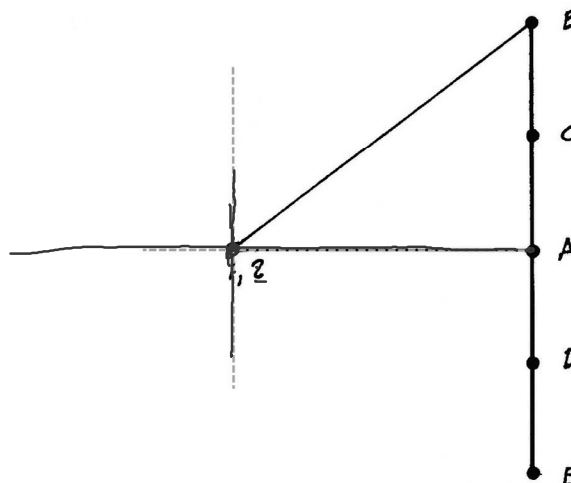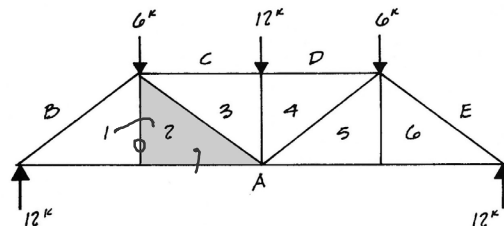
Draw the member forces

- Start in the external space (A) and cross to the first adjacent internal space (1).

- Draw the line you cross starting at the external letter (A).

- Now move to the next external space (B) adjacent to the same internal space (1).

- Draw a line with the same slope as the member crossed starting at the letter of the external space (B).

- These two lines cross at a point (1).

- Note that you needed 2 lines to define the location of the point (1).
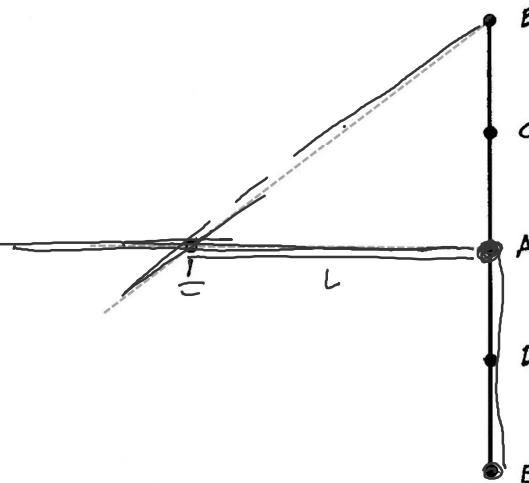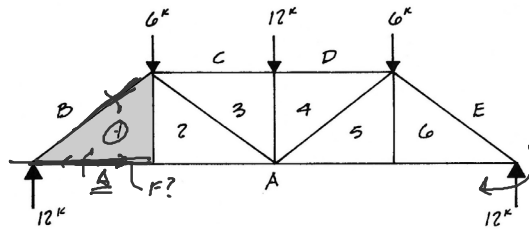
Graphic Statics - Step 5
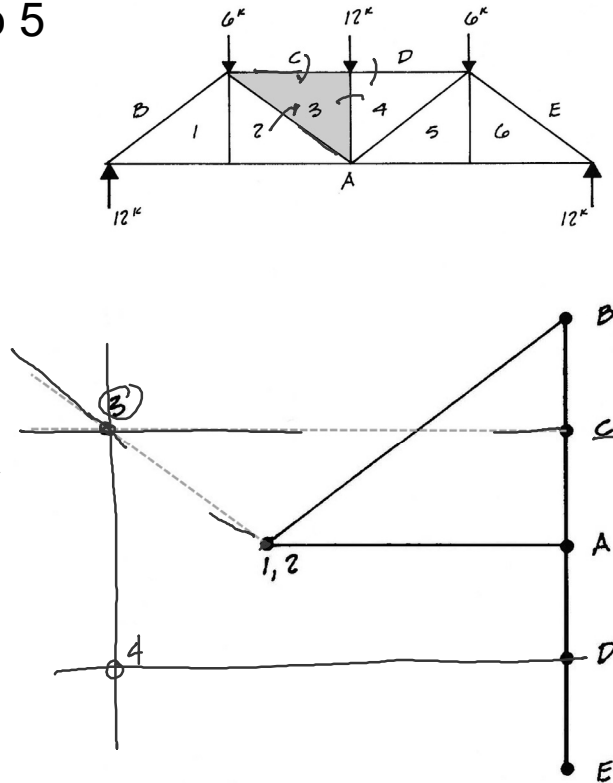
Draw the member forces

- Now find the next point (2).

- Choose two spaces that are adjacent to space this next space (2).

- Now move to the next external space (A) adjacent to this internal space (2).

- Draw a line with the same slope as the member crossed starting at the letter of the external space (A).

- Now observe the member which is between the next two spaces (1 & 2).

- Draw a line with the same slope as the member through the location of the point which labels the adjacent space (1).

- The scaled length of the line equals the force. If a line has zero length (like 1-2) then the force in the member is zero.

Graphic Statics - Step 5
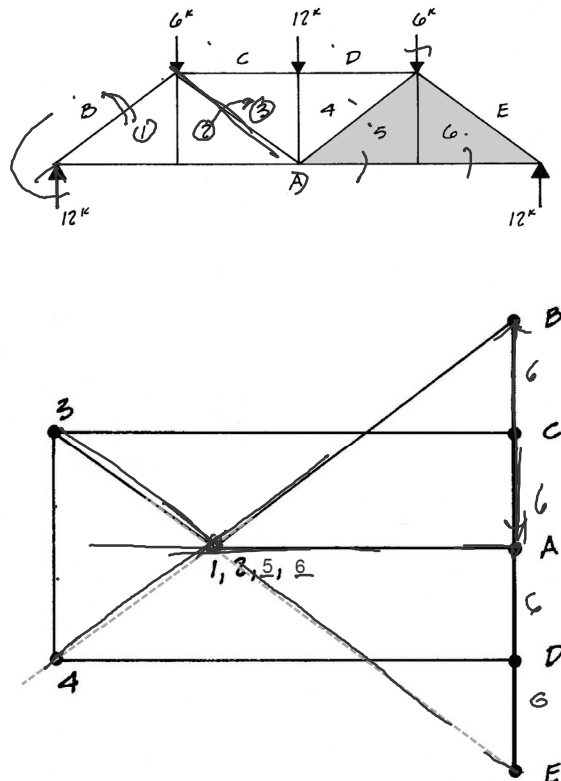
Draw the member forces

- Now find the next point (3).

- Choose two spaces that are adjacent to space this next space (3) which have points on the graph. (C&2)

- Find the member between the first pair of spaces (C&3). Draw a line with the same slope passing through the known point (C).

- Draw a second line with the same slope as the member crossed by the second pair (2&3) passing through the known point of the pair (2).

- Again, where these lines cross the unknown point can be found (3)

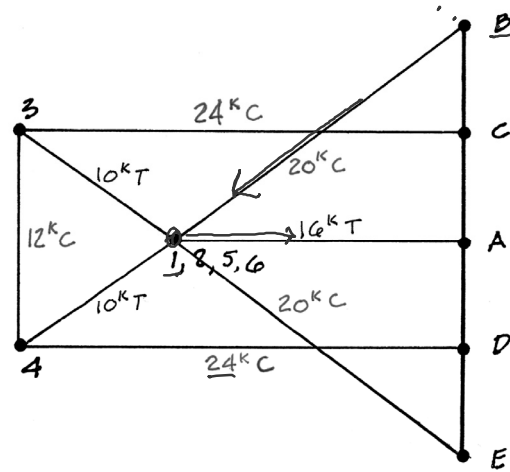
Graphic Statics - Step 5

Draw the member forces

- Continue moving across the truss until all points of the interior spaces are determined

Graphic Statics - Step 5

Draw the member forces

- Each line on the graph represents the scaled magnitude and direction of the force within it.

- to determine the sign of the force: Choose a member on the truss. Then choose a joint connected to that member. Moving about the joint in a clockwise direction note the space before and after crossing the member. This gives the direction of the force on the graphic diagram. Applying this direction to the FBD of the joint shows the sign: compression pushing and tension pulling.
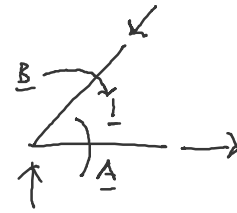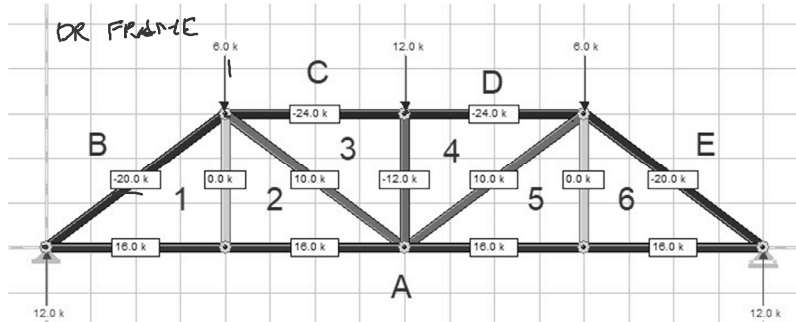
Graphic Statics - <u>Design</u>

The geometry and force diagrams are <u>duals</u>. Changing one will result in changes in the other and vice versa. Design a truss to achieve desired forces by modifying the force diagram, and then modifying the geometry diagram to match.
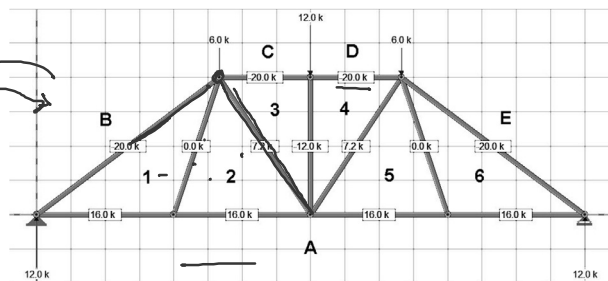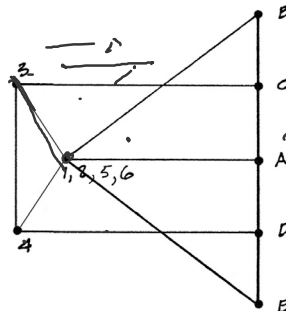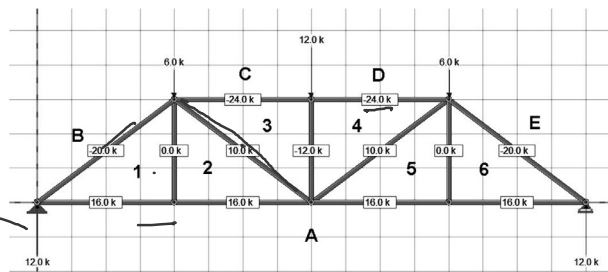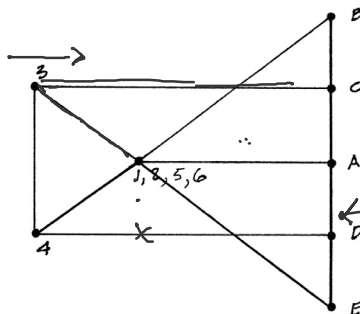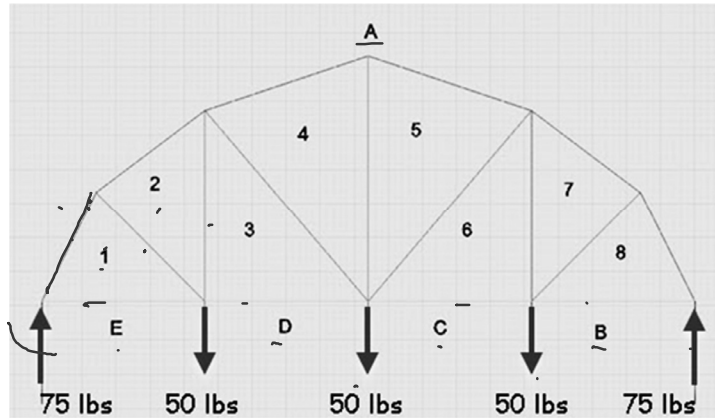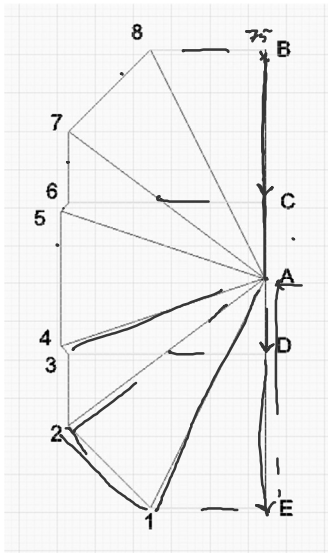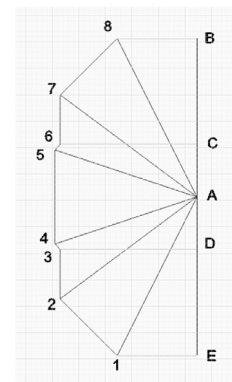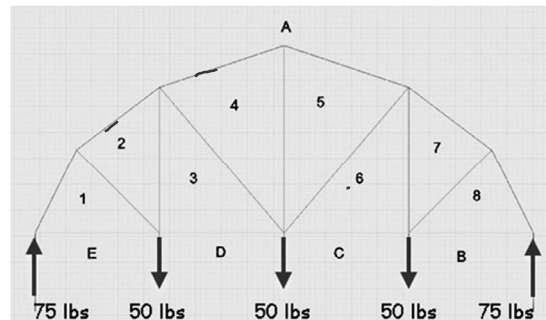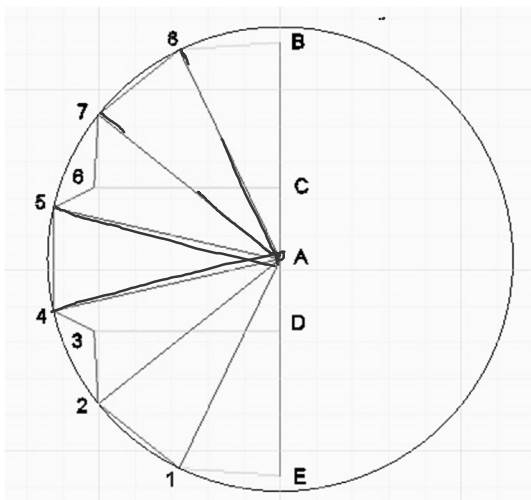
Truss Design

Start with a force diagram

- Draw the starting geometry
- Construct the force diagram as described above

Truss Design

Modify the force diagram

- Adjust the lengths of the members to alter the forces
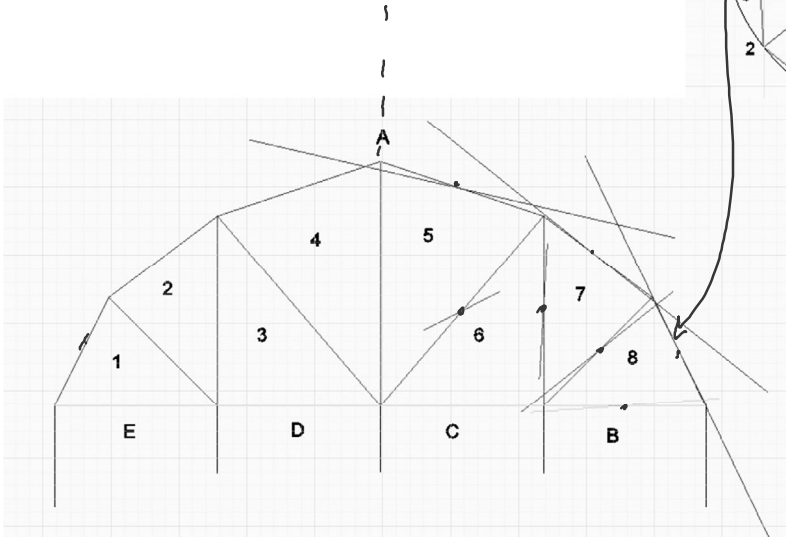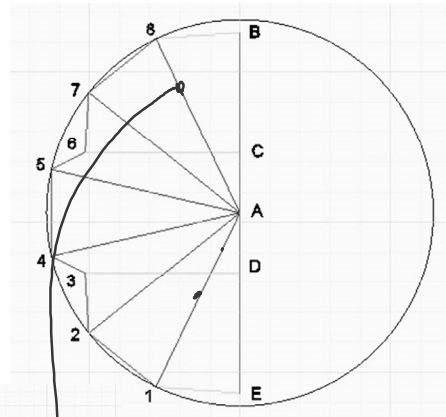- Retain the same topology (connectivity)

CHÒNGE

Truss Design
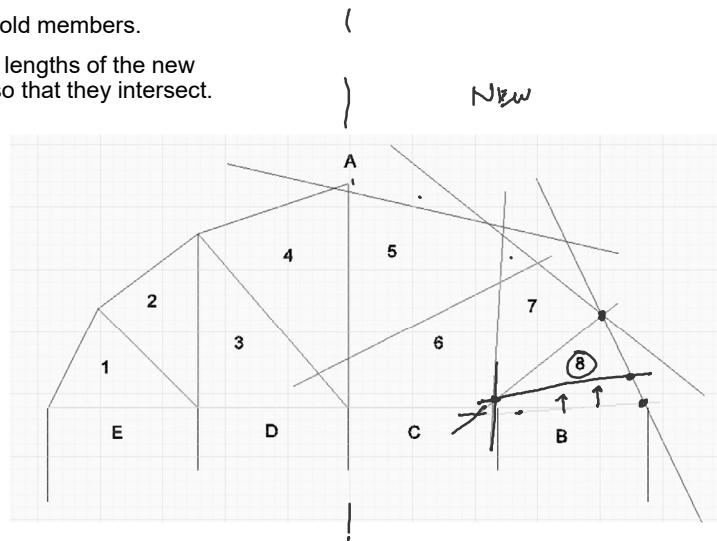
Change the slopes of the geometry

- Copy the members from the force diagram over the old members.

- Placing them at the mid-points of the old members is a good start.

Truss Design

Extend the new members

- Delete the old members.

- Extend the lengths of the new members so that they intersect.

New

- Since this truss is symmetric about the center line, only half of the geometry need be adjusted and then mirrored.
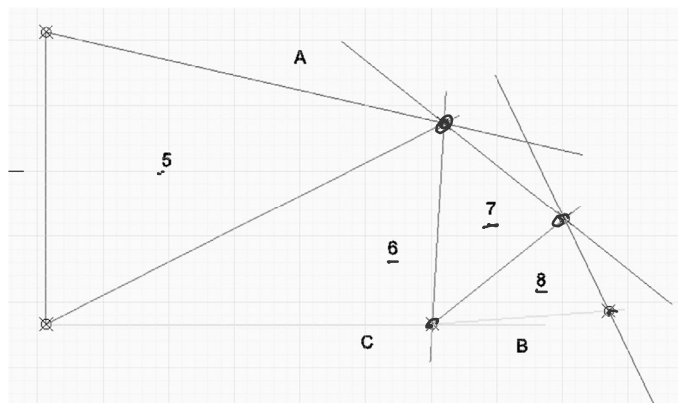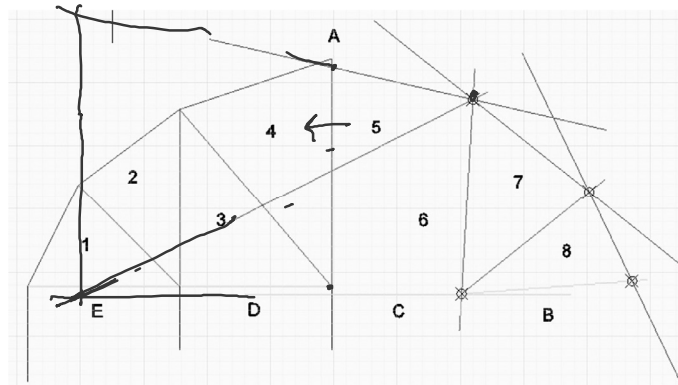
Truss Design

Form the first cell

- Move the members in the first cell so that all members pass through the three corner points of the cell

- Be carful not to move single points. Move whole members so that the slope does not change.

- Extent members as necessary to intersect members.

- When all members are concurrent at a node, mark the node with a point.

- Try to work across the truss one cell at a time.
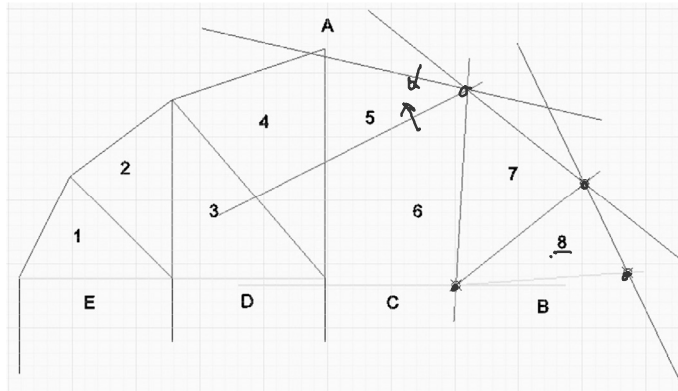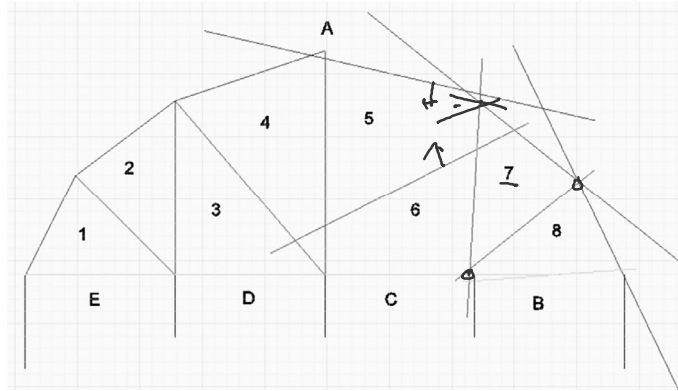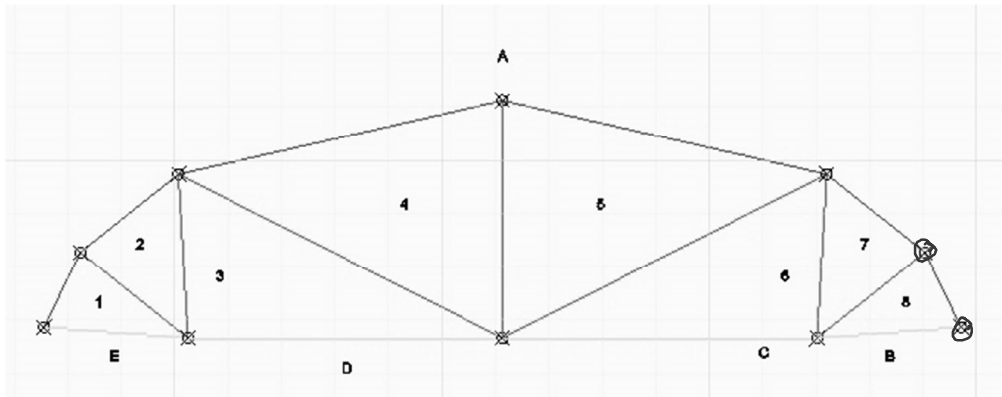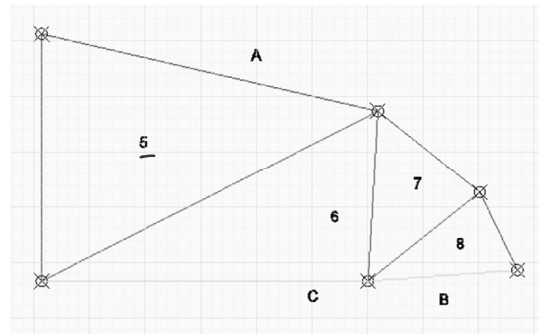
Truss Design

Form the next cells

- Continue to move across the truss one node at a time.

- In this case only half of the truss needed to be solved due to symmetry.
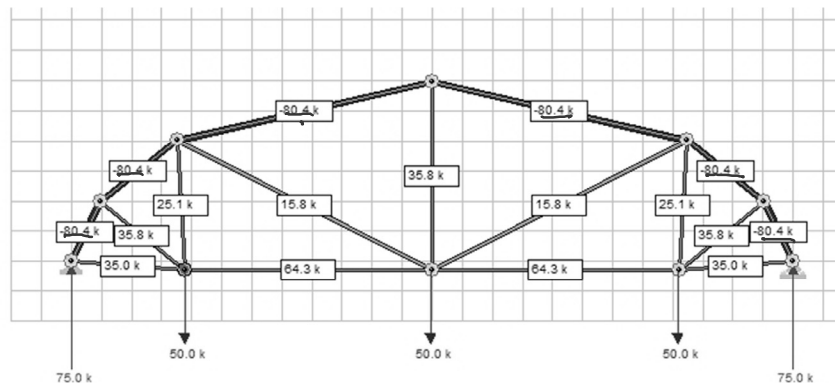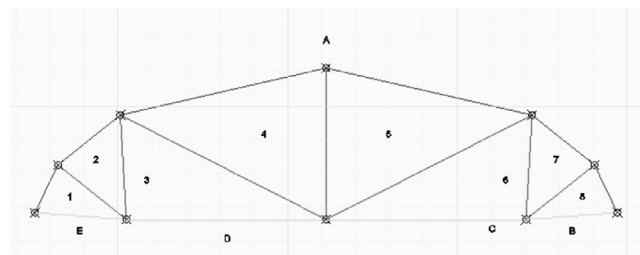
Truss Design

Form the final truss

- Clean up each node by trimming members
- Mirror the half solution to obtain the full truss.
- Scale the width to the length of the original span.

Truss Design
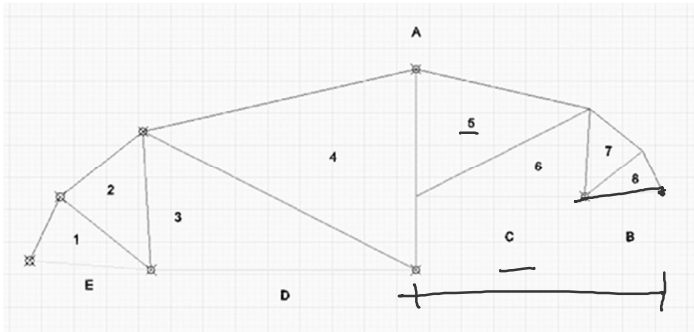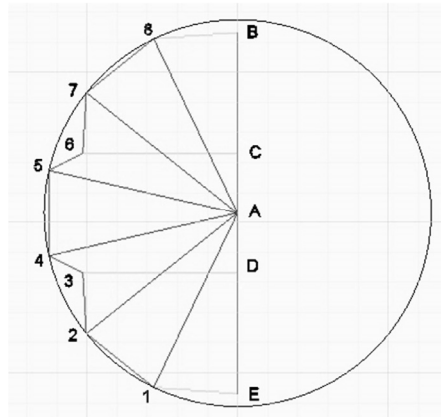
Check the final solution in Dr Frame

- Draw the geometry node by node in Dr Frame. Click a node and enter new coordinates.
- Check that the forces are as expected. Size the members.

Truss Design

Other solutions

- Taking a different starting point should yield the same geometry in the end.

Truss Design

Other solutions

- Additional constraints can be added to the geometry by altering the force diagram. For example a straight bottom chord.