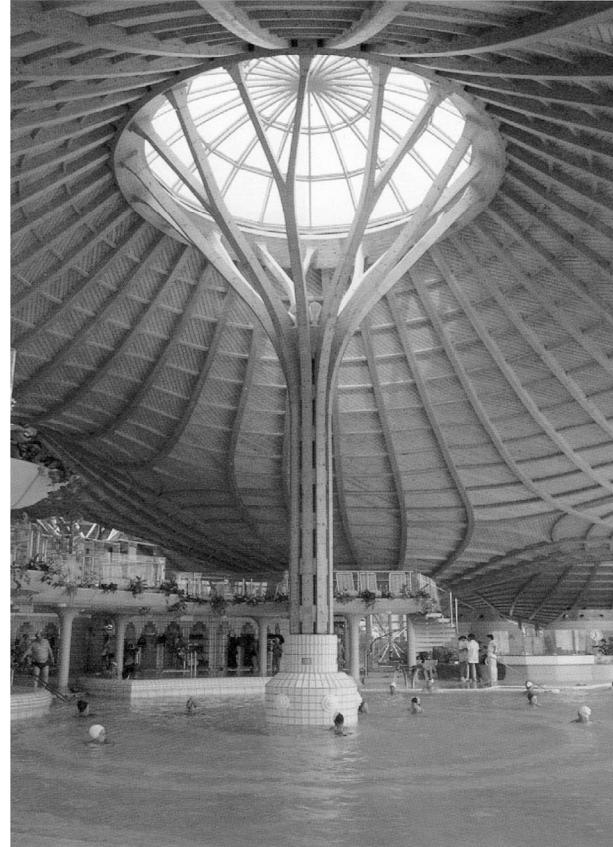


## Wood Columns

- Failure Modes
- Euler Equation
- End Conditions and Lateral Bracing
- Analysis of Wood Columns
- Design of Wood Columns



Solemar, Bad Dürkheim  
Klaus Linkwitz, 1987

## Failure Modes

**FLEXURE**

**Strength**

$$f_b = \frac{Mc}{I} \quad f_v = \frac{VQ}{Ib}$$

**AXIAL**

$$f_c = \frac{P}{A}$$

**Stability**

$$C_L = \frac{1 + (F_{bE}/F_b^*)}{1.9} - \sqrt{\left[ \frac{1 + (F_{bE}/F_b^*)}{1.9} \right]^2 - \frac{F_{bE}/F_b^*}{0.95}}$$

$$C_P = \frac{1 + (F_{cE}/F_c^*)}{2c} - \sqrt{\left[ \frac{1 + (F_{cE}/F_c^*)}{2c} \right]^2 - \frac{F_{cE}/F_c^*}{c}}$$

**Serviceability**

**Deflection**

**Bearing (crushing limit)**

## Leonhard Euler (1707 – 1783)

Euler Buckling (elastic buckling)

$$P_{cr} = \frac{\pi^2 AE}{\left(\frac{KL}{r}\right)^2} = \frac{\pi^2 IE}{(KL)^2}$$

$$r = \sqrt{\frac{I}{A}}$$

$$I = Ar^2$$

- A = Cross sectional area (in<sup>2</sup>)
- E = Modulus of elasticity of the material (lb/in<sup>2</sup>)
- K = Stiffness (curvature mode) factor
- L = Column length between pinned ends (in.)
- le = K L
- r = radius of gyration (in.)

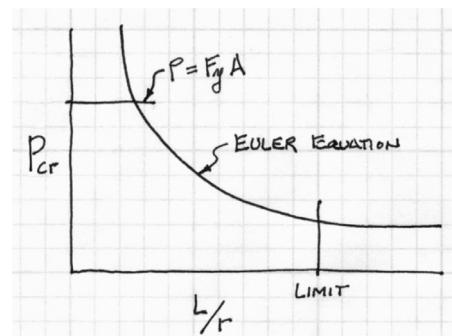
$$f_{cr} = \frac{\pi^2 E}{\left(\frac{KL}{r}\right)^2}$$

$$F_{cE} = \frac{0.822 E'_{min}}{\left(\frac{le}{d}\right)^2}$$

$$r = d/\sqrt{12}$$



portrait by Emanuel Handmann, 1753



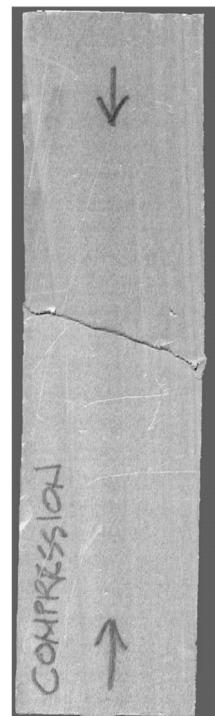
## Failure Mode - Strength

**Short Columns** – fail by crushing

$$f_c = \frac{P}{A} \leq F_c$$

$$A = \frac{P}{F_c}$$

- $f_c$  = Actual compressive stress
- A = Cross-sectional area of column (in<sup>2</sup>)
- P = Load on the column
- $F_c$  = Allowable compressive stress per codes



# Failure Modes – Stability

**Long Columns** – fail by buckling

Traditional Euler

$$f_{cr} = \frac{\pi^2 E}{\left(\frac{KL}{r}\right)^2}$$

- E = Modulus of elasticity of the column material (psi)
- K = Stiffness (curvature mode) factor
- L = Column length between ends (inches)
- r = radius of gyration =  $\sqrt{I/A}$  (inches)



NDS Equation

$$F_{cE} = \frac{0.822 E'_{min}}{\left(\frac{l_e}{d}\right)^2}$$

- $E'_{min}$  = reduced E modulus (psi)
- $l_e$  = Ke l (inches)
- d (inches)
- 0.822 =  $\pi^2/12$

$$r = d/\sqrt{12}$$

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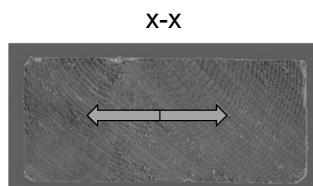
## Slenderness Ratio $l_e/d$

### Slenderness Ratios:

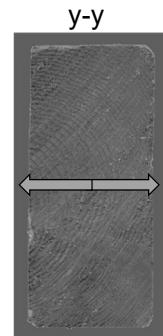
The larger ratio will govern.

Try to balance for efficiency.

Slenderness Limited to < 50



$$d = 3.5$$



$$b = 1.5$$

ratios for an 8 ft long 2x4:

$$X-X$$

$$K_e = 1.0$$

$$l_e = 1.0(96)$$

$$\frac{l_e}{d} = \frac{96}{3.5} = 27.4$$

$$Y-Y$$

$$K_e = 1.0$$

$$l_e = 1.0(96)$$

$$\frac{l_e}{b} = \frac{96}{1.5} = 64$$



# End Support Conditions

NDS 3.7.1.2

$K_e$  is a constant based on the end conditions

$l$  is the actual length

$l_e$  is the effective length (curved part)

$$l_e = K_e l$$

**Table G1 Buckling Length Coefficients,  $K_e$**

Buckling modes						
Theoretical $K_e$ value	0.5	0.7	1.0	1.0	2.0	2.0
Recommended design $K_e$ when ideal conditions approximated	0.65	0.80	1.2	1.0	2.10	2.4
End condition code						
	Rotation fixed, translation fixed					
	Rotation free, translation fixed					
	Rotation fixed, translation free					
	Rotation free, translation free					

use these →

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## Allowable Flexure Stress $F_c'$

$F_c$  from tables determined by species and grade

$F_c' = F_c$  (adjustment factors)

## Actual Flexure Stress $f_b$

$$f_b = P/A$$

$$F_c' \geq f_b$$

**Table 4A Base Design Values for Visually Graded Dimension Lumber (2"-4" thick)<sup>1,2</sup>**

(Cont.) (All species except Southern Pine — see Table 4B) (Tabulated design values are for normal load duration and dry service conditions. See NDS 4.3 for a comprehensive description of design value adjustment factors.)

### USE WITH TABLE 4A ADJUSTMENT FACTORS

Species and commercial grade	Size classification	Design values in pounds per square inch (psi)						Grading Rules Agency
		Bending $F_b$	Tension parallel to grain $F_t$	Shear parallel to grain $F_v$	Compression perpendicular to grain $F_{c\perp}$	Compression parallel to grain $F_c$	Modulus of Elasticity $E$	
<b>EASTERN HEMLOCK-BALSAM FIR</b>								
Select Structural		1250	575	140	335	1200	1,200,000	
No.1		775	350	140	335	1000	1,100,000	
No.2	2" & wider	575	275	140	335	825	1,100,000	
No.3		350	150	140	335	475	900,000	
Stud	2" & wider	450	200	140	335	525	900,000	
Construction		675	300	140	335	1050	1,000,000	
Standard	2"-4" wide	375	175	140	335	850	900,000	
Utility		175	75	140	335	550	800,000	
NELMA NSLB								

## Adjustment Factors

**Table 4.3.1 Applicability of Adjustment Factors for Sawn Lumber**

	ASD only	Load Duration Factor	ASD and LRFD									LRFD only			
			Wet Service Factor	Temperature Factor	Beam Stability Factor	Size Factor	Flat Use Factor	Incising Factor	Repetitive Member Factor	Column Stability Factor	Buckling Stiffness Factor	K <sub>F</sub>	Format Conversion Factor	Resistance Factor	Time Effect Factor
$F_b' = F_b$	x	C <sub>D</sub>	C <sub>M</sub>	C <sub>t</sub>	C <sub>L</sub>	C <sub>F</sub>	C <sub>fu</sub>	C <sub>i</sub>	C <sub>r</sub>	-	-	-	2.54	0.85	$\lambda$
$F_t' = F_t$	x	C <sub>D</sub>	C <sub>M</sub>	C <sub>t</sub>	-	C <sub>F</sub>	-	C <sub>i</sub>	-	-	-	-	2.70	0.80	$\lambda$
$F_v' = F_v$	x	C <sub>D</sub>	C <sub>M</sub>	C <sub>t</sub>	-	-	-	C <sub>i</sub>	-	-	-	-	2.88	0.75	$\lambda$
$F_c' = F_c$	x	C <sub>D</sub>	C <sub>M</sub>	C <sub>t</sub>	-	C <sub>F</sub>	-	C <sub>i</sub>	-	C <sub>P</sub>	-	-	2.40	0.90	$\lambda$
$F_{c\perp}' = F_{c\perp}$	x	-	C <sub>M</sub>	C <sub>t</sub>	-	-	-	C <sub>i</sub>	-	-	-	C <sub>b</sub>	1.67	0.90	-
$E' = E$	x	-	C <sub>M</sub>	C <sub>t</sub>	-	-	-	C <sub>i</sub>	-	-	-	-	-	-	-
$E_{min}' = E_{min}$	x	-	C <sub>M</sub>	C <sub>t</sub>	-	-	-	C <sub>i</sub>	-	-	C <sub>T</sub>	-	1.76	0.85	-

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### Allowable Flexure Stress $F_c'$

$F_c'$  from tables determined by species and grade

$$F_c' = F_c (C_D C_M C_t C_F C_i C_P)$$

Adjustment factors for compression:

C<sub>D</sub> Load Duration Factor

C<sub>t</sub> Temperature Factor

**Table 2.3.2 Frequently Used Load Duration Factors, C<sub>D</sub><sup>1</sup>**

Load Duration	C <sub>D</sub>	Typical Design Loads
Permanent	0.9	Dead Load
Ten years	1.0	Occupancy Live Load
Two months	1.15	Snow Load
Seven days	1.25	Construction Load
Ten minutes	1.6	Wind/Earthquake Load
Impact <sup>2</sup>	2.0	Impact Load

**Table 2.3.3 Temperature Factor, C<sub>t</sub>**

Reference Design Values	In-Service Moisture Conditions <sup>1</sup>	C <sub>t</sub>		
		T ≤ 100°F	100°F < T ≤ 125°F	125°F < T ≤ 150°F
F <sub>b</sub> , E, E <sub>min</sub>	Wet or Dry	1.0	0.9	0.9
F <sub>b</sub> , F <sub>v</sub> , F <sub>c</sub> , and F <sub>c⊥</sub>	Dry	1.0	0.8	0.7
	Wet	1.0	0.7	0.5

1. Wet and dry service conditions for sawn lumber, structural glued laminated timber, prefabricated wood I-joints, structural composite lumber, wood structural panels and cross-laminated timber are specified in 4.1.4, 5.1.4, 7.1.4, 8.1.4, 9.3.3, and 10.1.5 respectively.

- (1) Actual stress due to (DL)  $\leq (0.9)$  (Design value)
- (2) Actual stress due to (DL+LL)  $\leq (1.0)$  (Design value)
- (3) Actual stress due to (DL+WL)  $\leq (1.6)$  (Design value)
- (4) Actual stress due to (DL+LL+SL)  $\leq (1.15)$  (Design value)
- (5) Actual stress due to (DL+LL+WL)  $\leq (1.6)$  (Design value)
- (6) Actual stress due to (DL+SL+WL)  $\leq (1.6)$  (Design value)
- (7) Actual stress due to (DL+LL+SL+WL)  $\leq (1.6)$  (Design value)

## Allowable Flexure Stress $F_c'$ (For Dimensioned Lumber)

$F_c'$  from tables determined by species and grade

$$F_c' = F_c (C_D C_M C_t C_F C_i C_P)$$

Adjustment factors for compression:

$C_M$  Moisture Factor

$C_F$  Size Factor

### Wet Service Factor, $C_M$

When dimension lumber is used where moisture content will exceed 19% for an extended time period, design values shall be multiplied by the appropriate wet service factors from the following table:

Wet Service Factors,  $C_M$

$F_b$	$F_t$	$F_v$	$F_{c\perp}$	$F_c$	E and $E_{min}$
0.85*	1.0	0.97	0.67	0.8**	0.9

\* when  $(F_b)(C_F) \leq 1,150$  psi,  $C_M = 1.0$

\*\* when  $(F_c)(C_F) \leq 750$  psi,  $C_M = 1.0$

Size Factors,  $C_F$

Grades	Width (depth)	$F_b$		$F_t$	$F_c$		
		Thickness (breadth)					
		2" & 3"	4"				
Select	2", 3", & 4"	1.5	1.5	1.5	1.15		
	5"	1.4	1.4	1.4	1.1		
	6"	1.3	1.3	1.3	1.1		
	8"	1.2	1.3	1.2	1.05		
	10"	1.1	1.2	1.1	1.0		
	12"	1.0	1.1	1.0	1.0		
	14" & wider	0.9	1.0	0.9	0.9		
Stud	2", 3", & 4"	1.1	1.1	1.1	1.05		
	5" & 6"	1.0	1.0	1.0	1.0		
	8" & wider	Use No.3 Grade tabulated design values and size factors					
Construction, Standard	2", 3", & 4"	1.0	1.0	1.0	1.0		
Utility	4"	1.0	1.0	1.0	1.0		
	2" & 3"	0.4	—	0.4	0.6		

## Allowable Flexure Stress $F_c'$ (For Timbers)

$F_c'$  from tables determined by species and grade

$$F_c' = F_c (C_D C_M C_t C_F C_i C_P)$$

Adjustment factors for compression:

$C_M$  Moisture Factor

$C_F$  Size Factor

### Size Factor, $C_F$

When visually graded timbers are subjected to loads applied to the narrow face, tabulated design values shall be multiplied by the following size factors:

Size Factors,  $C_F$

Depth	$F_b$	$F_t$	$F_c$
$d > 12"$	$(12/d)^{1/9}$	1.0	1.0
$d \leq 12"$	1.0	1.0	1.0

### Wet Service Factor, $C_M$

When timbers are used where moisture content will exceed 19% for an extended time period, design values shall be multiplied by the appropriate wet service factors from the following table (for Southern Pine and Mixed Southern Pine, use tabulated design values without further adjustment):

Wet Service Factors,  $C_M$

$F_b$	$F_t$	$F_v$	$F_{c\perp}$	$F_c$	E and $E_{min}$
1.00	1.00	1.00	0.67	0.91	1.00

## Allowable Flexure Stress $F_c'$

$F_c$  from tables determined by species and grade

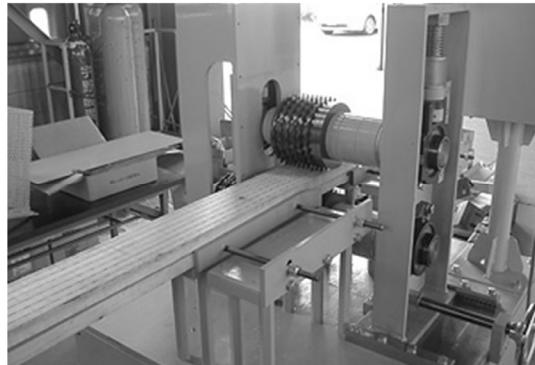
$$F_c' = F_c (C_D C_M C_t C_F C_i C_P)$$

Adjustment factors for compression :

$C_i$  Incising Factor

**Table 4.3.8 Incising Factors,  $C_i$**

Design Value	$C_i$
$E, E_{min}$	0.95
$F_b, F_t, F_c, F_v$	0.80
$F_{cl}$	1.00



## Allowable Flexure Stress $F_c'$

$F_c$  from tables determined by species and grade

$$F_c' = F_c (C_D C_M C_t C_F C_i C_P)$$

## 3.7 Solid Columns

### 3.7.1 Column Stability Factor, $C_p$

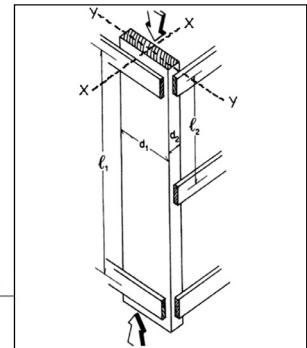
3.7.1.1 When a compression member is supported throughout its length to prevent lateral displacement in all directions,  $C_p = 1.0$ .

3.7.1.2 The effective column length,  $\ell_e$ , for a solid column shall be determined in accordance with principles of engineering mechanics. One method for determining effective column length, when end-fixity conditions are known, is to multiply actual column length by the appropriate effective length factor specified in Appendix G,  $\ell_e = (K_e)(\ell)$ .

3.7.1.3 For solid columns with rectangular cross section, the slenderness ratio,  $\ell_e/d$ , shall be taken as the larger of the ratios  $\ell_{e1}/d_1$  or  $\ell_{e2}/d_2$  (see Figure 3F) where each ratio has been adjusted by the appropriate buckling length coefficient,  $K_e$ , from Appendix G.

3.7.1.4 The slenderness ratio for solid columns,  $\ell_e/d$ , shall not exceed 50 except that during construction  $\ell_e/d$  shall not exceed 75.

3.7.1.5 The column stability factor shall be calculated as follows:



$$C_p = \frac{1 + (F_{ce}/F_c)}{2c} - \sqrt{\left[ \frac{1 + (F_{ce}/F_c)}{2c} \right]^2 - \frac{F_{ce}/F_c}{c}} \quad (3.7-1)$$

where:

$F_c^*$  = reference compression design value parallel to grain multiplied by all applicable adjustment factors except  $C_p$  (see 2.3), psi

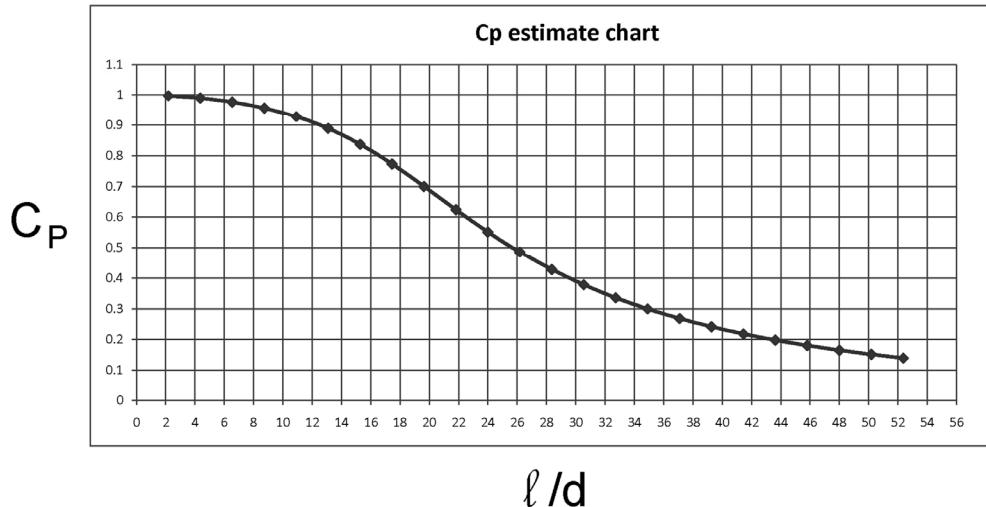
$$F_{ce} = \frac{0.822 E_{min}'}{(\ell_e/d)^2}$$

$c = 0.8$  for sawn lumber

$c = 0.85$  for round timber poles and piles

$c = 0.9$  for structural glued laminated timber or structural composite lumber

## $C_P$ estimation



## Analysis of Wood Columns

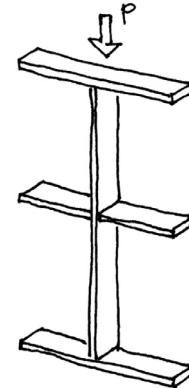
### Data:

- Column – size, length
- Support conditions
- Material properties –  $F_c$ , E
- Load

### Required:

- Pass/Fail or margin of safety

- Calculate slenderness ratio  $\ell_e/d$   
largest ratio governs. Must be < 50
- Find adjustment factors (all except  $C_P$ )  
 $C_D$   $C_M$   $C_t$   $C_F$   $C_i$
- Calculate  $C_P$
- Determine  $F'_c$  by multiplying the tabulated  $F_c$  by all the above factors
- Calculate the actual stress:  $f_c = P/A$
- Compare Allowable and Actual stress.  
 $F'_c > f_c$  passes



## Analysis Example:

Data: section 4x8 (nominal)  
 Douglas Fir-Larch No1  
 M.C. 15%  
 P = 7000 LBS (Snow Load)

Find: Pass/Fail

From NDS Supplement Table 4A

$$F_c = 1500 \text{ psi}$$

$$E_{min} = 620000 \text{ psi}$$

$$C_D = 1.15 \text{ (snow)}$$

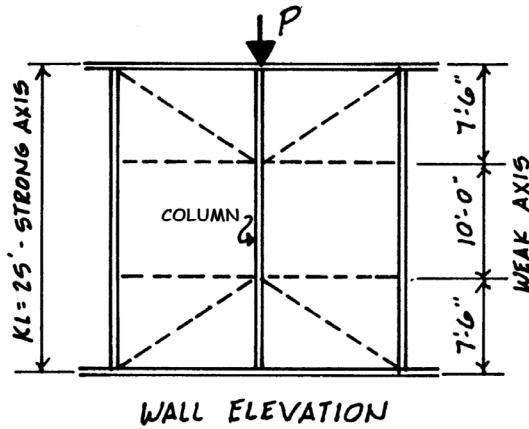
$$C_M = 1.0$$

$$C_t = 1.0$$

$$C_F = 1.05 \text{ (4x8)}$$

$$C_i = 1.0$$

$$C_P = ?$$



Size Factors,  $C_F$

		$F_c$
Grades	Width (depth)	
Select	2", 3", & 4"	1.15
	5"	1.1
	6"	1.1
	8"	1.05
	10"	1.0
	12"	1.0
	14" & wider	0.9
Stud	2", 3", & 4"	1.05
	5" & 6"	1.0
	8" & wider	
Construction, Standard	2", 3", & 4"	1.0
Utility	4"	1.0
	2" & 3"	0.6

## Analysis Example:

Calculate  $C_P$

$$C_p = \frac{1 + (F_{cE}/F_c^*)}{2c} - \sqrt{\left[ \frac{1 + (F_{cE}/F_c^*)}{2c} \right]^2 - \frac{F_{cE}/F_c^*}{c}} \quad (3.7-1)$$

where:

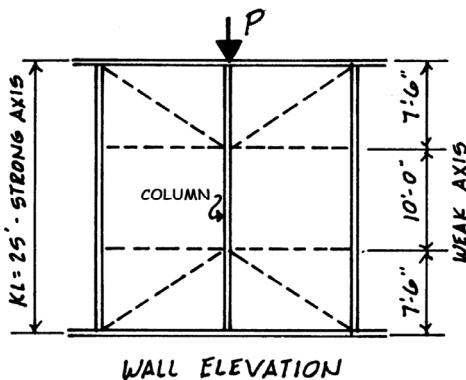
$F_c^*$  = reference compression design value parallel to grain multiplied by all applicable adjustment factors except  $C_p$  (see 2.3), psi

$$F_{cE} = \frac{0.822 E_{min}}{(\ell_e/d)^2}$$

c = 0.8 for sawn lumber

c = 0.85 for round timber poles and piles

c = 0.9 for structural glued laminated timber or structural composite lumber



$$\begin{aligned} \ell_{ex} &= 25' = 300'' & \ell_{ey} &= 10' = 120'' \\ \ell_{ex}/d_1 &= \frac{300''}{7.25''} & \ell_{ey}/d_2 &= \frac{120''}{3.5''} \\ &= 41.4 & &= 34.3 \end{aligned}$$

$$\ell_e/d = 41.4 < 50 \quad \checkmark$$

## Analysis Example:

Calculate  $C_p$

$$C_p = \frac{1 + (F_{ce}/F_c^*)}{2c} - \sqrt{\left[ \frac{1 + (F_{ce}/F_c^*)}{2c} \right]^2 - \frac{F_{ce}/F_c^*}{c}} \quad (3.7-1)$$

where:

$F_c^*$  = reference compression design value parallel to grain multiplied by all applicable adjustment factors except  $C_p$  (see 2.3), psi

$$F_{ce} = \frac{0.822 E'_{min}}{(\ell_e/d)^2}$$

c = 0.8 for sawn lumber

c = 0.85 for round timber poles and piles

c = 0.9 for structural glued laminated timber or structural composite lumber

$$\begin{aligned} F_{ce} &= \frac{0.822 E'_{min}}{(l_e/d)^2} \\ &= \frac{0.822 (420000)}{(41.4)^2} \\ &= 297.6 \text{ psi} \end{aligned}$$

$$\begin{aligned} F_c^* &= 1500 (1.15 \ 1.05) \\ &= 1811.25 \text{ psi} \end{aligned}$$

$$F_{ce}/F_c^* = \frac{297.6}{1811.25} = 0.164$$

$$c = 0.8$$

## Analysis Example:

Calculate  $C_p$

$$C_p = \frac{1 + (F_{ce}/F_c^*)}{2c} - \sqrt{\left[ \frac{1 + (F_{ce}/F_c^*)}{2c} \right]^2 - \frac{F_{ce}/F_c^*}{c}} \quad (3.7-1)$$

where:

$F_c^*$  = reference compression design value parallel to grain multiplied by all applicable adjustment factors except  $C_p$  (see 2.3), psi

$$F_{ce} = \frac{0.822 E'_{min}}{(\ell_e/d)^2}$$

c = 0.8 for sawn lumber

c = 0.85 for round timber poles and piles

c = 0.9 for structural glued laminated timber or structural composite lumber

$$C_p = \frac{1 + 0.164}{2(0.8)} - \sqrt{\left[ \frac{1 + 0.164}{2(0.8)} \right]^2 - \frac{0.164}{0.8}}$$

$$C_p = 0.1584$$

$$\begin{aligned} F_c^* &= 1500 (1.15 \ 1.05 \ 0.1584) \\ &= 286.9 \text{ psi} \end{aligned}$$

$$f_c = \frac{P}{A} = \frac{7000}{25.38 \text{ in}^2} = 275.8 \text{ psi}$$

$$F_c^* > f_c \quad \checkmark \text{ OK}$$

Compare Allowable and Actual stress  
 $F'_c > f_c$  passes

# Capacity Analysis of Columns

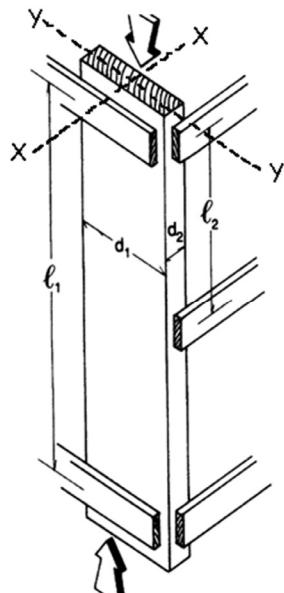
## Data:

- Column – size, length
- Support conditions
- Material properties –  $F_c$ , E

## Required:

- Maximum Load,  $P_{max}$

- Calculate slenderness ratio  $\ell_e/d$   
largest ratio governs. Must be < 50
- Find adjustment factors (all except  $C_p$ )  
 $C_D C_M C_t C_F C_i$
- Calculate  $C_p$
- Determine  $F'_c$  by multiplying the tabulated  $F_c$  by all the above factors
- Set actual stress = allowable,  $f_c = F'_c$
- Find the maximum allowable load  
 $P_{max} = F'_c A$



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# Capacity Example

## Data:

- 4x10
- Hem – Fir, No 2 M.C. = 20%
- Wind Load
- $L_1 = 8'$   $L_2 = 4'$   $K_e = 1.0$

## Required:

- Maximum Load,  $P_{max}$

From NDS Supplement Table 4A

$$F_c = 1300 \text{ psi}$$

$$E_{min} = 470000 \text{ psi}$$

$$C_D = 1.6$$

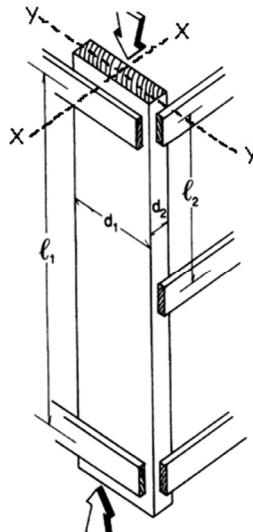
$$C_{Mc} = 0.8 \quad C_{ME} = 0.9$$

$$C_t = 1.0$$

$$C_F = 1.0$$

$$C_i = 1.0$$

$$C_p = ?$$



$$\begin{aligned} X-X \\ \ell_{ex} &= 8' = 96'' \\ \frac{\ell_{ex}}{d_1} &= \frac{96}{9.25} = 10.4 \end{aligned}$$

$$\begin{aligned} Y-Y \\ \ell_{ey} &= 4' = 48'' \\ \frac{\ell_{ey}}{d_2} &= \frac{48}{3.5} = 13.7 \end{aligned}$$

$$\frac{\ell_e}{d} = 13.7 < 50 \quad \checkmark$$

## Allowable Flexure Stress $F_c'$

$4 \times 10 \text{ M.C } 20\% \quad F_c = 1300 \text{ psi}$

$F_c'$  from tables determined by species and grade

$$F_c' = F_c (C_D C_M C_t C_F C_i C_P)$$

Adjustment factors for compression:

$C_M$  Moisture Factor

$C_F$  Size Factor

## Wet Service Factor, $C_M$

When dimension lumber is used where moisture content will exceed 19% for an extended time period, design values shall be multiplied by the appropriate wet service factors from the following table:

Wet Service Factors,  $C_M$

$F_b$	$F_t$	$F_v$	$F_{cl}$	$F_c$	E and $E_{min}$
0.85*	1.0	0.97	0.67	0.8**	0.9

\* when  $(F_b)(C_F) \leq 1,150 \text{ psi}$ ,  $C_M = 1.0$

\*\* when  $(F_c)(C_F) \leq 750 \text{ psi}$ ,  $C_M = 1.0$

Size Factors,  $C_F$

Grades	Width (depth)	$F_b$		$F_t$	$F_c$
		Thickness (breadth)			
Select	2", 3", & 4"	1.5	1.5	1.5	1.15
	5"	1.4	1.4	1.4	1.1
	6"	1.3	1.3	1.3	1.1
	8"	1.2	1.3	1.2	1.05
	10"	1.1	1.2	1.1	1.0
	12"	1.0	1.1	1.0	1.0
	14" & wider	0.9	1.0	0.9	0.9
Stud	2", 3", & 4"	1.1	1.1	1.1	1.05
	5" & 6"	1.0	1.0	1.0	1.0
	8" & wider	Use No.3 Grade tabulated design values and size factors			
Construction, Standard	2", 3", & 4"	1.0	1.0	1.0	1.0
Utility	4"	1.0	1.0	1.0	1.0
	2" & 3"	0.4	—	0.4	0.6

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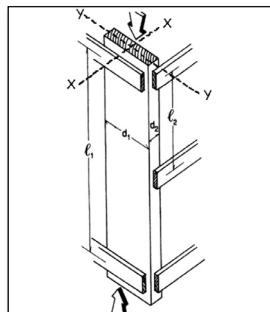
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## Capacity Example

Find  $C_p$

$$\begin{aligned} F_{ce} &= \frac{0.822 E'_{min}}{(\ell_e/d)^2} \\ &= \frac{0.822(470000)(0.9)}{13.7^2} \\ &= 1848.7 \text{ psi} \end{aligned}$$

$$\begin{aligned} F_c^* &= 1300(1.6 \quad 0.8) \\ &= 1664. \text{ psi} \end{aligned}$$



$$C_p = \frac{1 + (F_{ce}/F_c^*)}{2c} - \sqrt{\left[ \frac{1 + (F_{ce}/F_c^*)}{2c} \right]^2 - \frac{F_{ce}/F_c^*}{c}} \quad (3.7-1)$$

where:

$F_c^*$  = reference compression design value parallel to grain multiplied by all applicable adjustment factors except  $C_p$  (see 2.3), psi

$$F_{ce} = \frac{0.822 E'_{min}}{(\ell_e/d)^2}$$

c = 0.8 for sawn lumber

c = 0.85 for round timber poles and piles

c = 0.9 for structural glued laminated timber or structural composite lumber

Find the maximum load,  $P_{max}$

$$F_{ce}/F_c^* = \frac{1848.7}{1664} = 1.11$$

$$\begin{aligned} F_c^* &= 1300(1.6 \quad 0.8 \quad 0.7261) \\ &= 1208 \text{ psi} \end{aligned}$$

$$P_{max} = F_c^* A = 1208(32.38) = 39115 \text{ *}$$

$$C_p = 0.7261$$

# Timber Column Design

## Given:

- Lumber species, grade
- Conditions of use
- Load

## Required:

- column size
- 1. Find adjustment factors (all except  $C_P$ )  
 $C_D C_M C_t C_F C_i$
- 2. Guess  $C_P$
- 3. Estimate Area and d (based on bracing)
- 4. Calculate slenderness ratio  $I_e/d$   
 largest ratio governs. Must be  $< 50$
- 5. Calculate  $C_P$
- 6. Determine  $F'_c$  by multiplying the tabulated  $F_c$  by all the above factors
- 7. Revise Area:  $A = P/F'_c$
- 8. Revise  $C_P$



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# Timber Column Design

## Given:

- White Oak, No.1
- dry use, normal temp., not incised
- Load:  $D+L=55$  psf

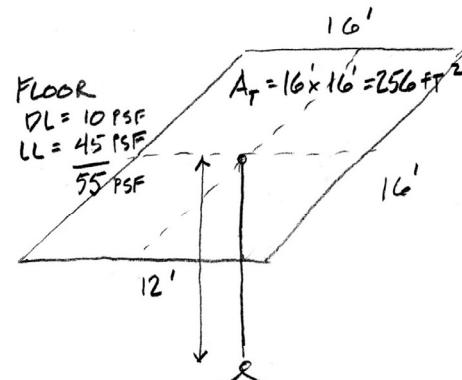
## Required:

- column size

1. Find adjustment factors (all except  $C_P$ )

$$C_D C_M C_t C_F C_i$$

2. Guess  $C_P \rightarrow$  try 0.5



**Table 4D Reference Design Values for Visually Graded Timbers (5" x 5" and larger)<sup>1,3</sup>**

(Tabulated design values are for normal load duration and dry service conditions, unless specified otherwise. See NDS 4.3 for a comprehensive description of design value adjustment factors.)

### USE WITH TABLE 4D ADJUSTMENT FACTORS

Species and commercial Grade	Size classification	Design values in pounds per square inch (psi)						Modulus of Elasticity E	E <sub>min</sub>	Specific Gravity <sup>4</sup> G	Grading Rules Agency
		Bending F <sub>b</sub>	Tension parallel to grain F <sub>t</sub>	Shear parallel to grain F <sub>v</sub>	Compression perpendicular to grain F <sub>c,L</sub>	Compression parallel to grain F <sub>c</sub>					
<b>WHITE OAK</b>											
Select Structural	Beams and Stringers	1,400	825	205	800	900	1,000,000	370,000	0.73	NELMA	
No.1		1,200	575	205	800	775	1,000,000	370,000			
No.2		750	375	205	800	475	800,000	290,000			
Select Structural	Posts and Timbers	1,300	875	205	800	950	1,000,000	370,000			
No.1		1,050	700	205	800	825	1,000,000	370,000			
No.2		600	400	205	800	400	800,000	290,000			

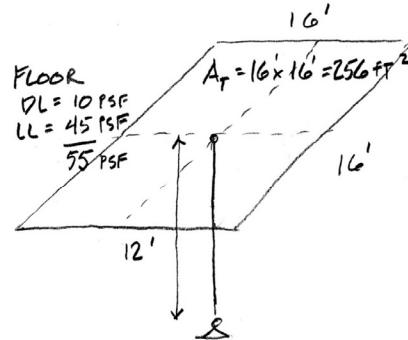
# Timber Column Design

## Given:

- White Oak, No.1
- dry use, normal temp., not incised
- Load: D+L=55 psf

## Required:

- column size



- Find adjustment factors (all except  $C_p$ )

$$C_D C_M C_t C_F C_i$$

- Guess  $C_p \rightarrow$  try 0.5
- Estimate Area and d (based on bracing)
- Calculate slenderness ratio  $l_e/d$

largest ratio governs. Must be < 50

ESTIMATE SIZE:

$$\text{GUESS } C_p = 0.5 \\ A = \frac{P}{F_c} = \frac{14080^*}{825(.5)} = 34 \text{ in}^2$$

TRY:

$$\sqrt{A} = d \quad \sqrt{34} = 5.8$$

say 5.5" x 5.5"

# Timber Column Design

## Given:

- White Oak, No.1
- dry use, normal temp., not incised
- Load: D+L=55 psf

TRY 6x6

$$\frac{l_e}{d} = \frac{(1)144}{5.5} = 26.18$$

CHECK  $C_p$  ON GRAPH - ADJUST IF NEEDED

$$F_{CE} = \frac{0.822(370000)}{26.18^2} = 443.7 \text{ psi}$$

$$F_c^* = F_c (C_D C_M C_F C_t C_i) = 825 \text{ psi}$$

$$F_{CE}/F_c^* = \frac{443.7}{825} = 0.5378$$

$$C_p = \frac{1 + 0.5378}{2(0.8)} - \sqrt{\left[ \frac{1 + 0.5378}{2(0.8)} \right]^2 - \frac{0.5378}{0.8}}$$

$$C_p = 0.46$$

