## Architecture 544

## Wood Columns

- Failure Modes
- Euler Equation
- End Conditions and Lateral Bracing
- Analysis of Wood Columns
- Design of Wood Columns



## Failure Modes

## FLEXURE

AXIAL

## Strength

$$
f_{b}=\frac{M c}{I} \quad f_{v}=\frac{V Q}{I b}
$$

$$
f_{c}=\frac{P}{A}
$$

## Stability



## Serviceability

Euler Buckling (elastic buckling)


$$
\begin{aligned}
& r=\sqrt{\frac{I}{A}} \\
& I=A r^{2}
\end{aligned}
$$

- A = Cross sectional area (in ${ }^{2}$ )
- $\quad E=$ Modulus of elasticity of the material ( $\mathrm{l} / \mathrm{i} / \mathrm{in}^{2}$ )
- $\quad \mathrm{K}=$ Stiffness (curvature mode) factor

- $\quad L=$ Column length between pinned ends (in.)
- $\quad \mathrm{le}=\mathrm{KL}$



## Failure Mode - Strength

Short Columns - fail by crushing

$$
f_{c}=\frac{P}{A} \leq F_{c} \quad A=\frac{P}{\underline{F_{c}}}
$$

- $\quad f_{c}=$ Actual compressive stress
- $\quad \mathrm{A}=$ Cross-sectional area of column (in ${ }^{2}$ )
- $P=$ Load on the column
- $\quad F_{c}=$ Allowable compressive stress per codes



## Failure Modes - Stability

## Long Columns - fail by buckling

Traditional Euler


- $E=$ Modulus of elasticity of the column material (psi)
- $\quad \mathrm{K}=$ Stiffness (curvature mode) factor
- $\quad L=$ Column length between ends (inches)
- $\quad r=$ radius of gyration $=\sqrt{\mathrm{I} / \mathrm{A}}$ (inches)

NDS Equation


- $\quad$ E'min $=$ reduced E modulus (psi)
- $\quad \ell \mathrm{e}=\mathrm{Ke} \ell$ (inches)
- d (inches)
$-0.822=\pi^{2} / 12$


## Slenderness Ratio $\ell_{\mathrm{e}} / \mathrm{d}$

## Slenderness Ratios:

The larger ratio will govern.
Try to balance for efficiency.
Slenderness Limited to < 50

$d=3.5$

$\mathrm{b}=1.5$
ratios for an 8 ft long $2 \times 4$ :

$$
\begin{array}{ll}
x-x & y-Y \\
K_{e}=1.0 & K_{e}=1.0 \\
l_{e}=1.0(96) & l_{e}=1.0(96) \\
\frac{l_{e}}{d}=\frac{96}{3.5}=27.4 & \frac{l_{e}}{b}=\frac{96}{1.5}=64>50
\end{array}
$$



## End Support Conditions

NDS 3.7.1.2
$\mathrm{K}_{\mathrm{e}}$ is a constant based on the end conditions
$\ell$ is the actual length
$\ell_{e}$ is the effective length (curved part)
$\ell=K_{e} \ell$

| Table G1 | Buckling Length Coefficients, $\mathrm{K}_{\mathbf{e}}$ |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | X |  |  |  |  |  |
| Buckling modes |  |  |  |  | $\underbrace{1}_{1} \begin{aligned} & 1 \\ & 1 \\ & 1 \\ & 1 \\ & 1 \\ & 1 \\ & 1 \end{aligned}$ | $\begin{gathered} 1 \\ \frac{1}{4} \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ k \\ \hline \end{gathered}$ |
| Theoretical $K_{e}$ value | 0.5 | 0.7 | 1.0 | 1.0 | 2.0 | 2.0 |
| Recommended design $K_{e}$ when ideal conditions approximated | 0.65 | 0.80 |  | 1.0 | 2.10 | 2.4 |
| End condition code |  | Rotation fixed, translation fixed <br> Rotation free, translation fixed <br> Rotation fixed, translation free <br> Rotation free, translation free |  |  |  |  |

## Allowable Flexure Stress $\mathrm{F}_{\mathrm{c}}{ }^{\prime}$

Actual Flexure Stress $\mathrm{f}_{\mathrm{b}}$
$\mathrm{F}_{\mathrm{c}}$ from tables determined by species and grade
$\mathrm{F}_{\mathrm{c}}{ }^{\prime}=\mathrm{F}_{\mathrm{c}}$ (adjustment factors)

$$
F_{c}{ }_{c} \geq f_{c}
$$

Table 4A Base Design Values for Visually Graded Dimension Lumber (2"-4" thick) ${ }^{\mathbf{1 , 2}}$ (Cont.)
(All species except Southern Pine - see Table 4B) (Tabulated design values are for normal load duration and dry service conditions. See NDS 4.3 for a comprehensive description of design value adjustment factors.)

USE WITH TABLE 4A ADJUSTMENT FACTORS


Adjustment Factors
Table 4.3.1 Applicability of Adjustment Factors for Sawn Lumber

|  |  |  | ASD and LRFD |  |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| $\mathrm{F}_{\mathrm{b}}{ }^{\prime}=\mathrm{F}_{\mathrm{b}}$ | x | $\mathrm{C}_{\mathrm{D}}$ | $\mathrm{C}_{\mathrm{M}}$ | $\mathrm{C}_{t}$ | $\mathrm{C}_{\mathrm{L}}$ | $\mathrm{C}_{\mathrm{F}}$ | $\mathrm{C}_{\mathrm{fu}}$ | $\mathrm{C}_{i}$ | $\mathrm{C}_{\mathrm{r}}$ | - | - | - | 2.54 | 0.85 | $\lambda$ |
| $\mathrm{F}_{\mathrm{t}}{ }^{\prime}=\mathrm{F}_{\mathrm{t}}$ | x | $\mathrm{C}_{\mathrm{D}}$ | $\mathrm{C}_{\mathrm{M}}$ | $\mathrm{C}_{t}$ | - | $\mathrm{C}_{\mathrm{F}}$ | - | $\mathrm{C}_{\mathrm{i}}$ | - | - | - | - | 2.70 | 0.80 | $\lambda$ |
| $\mathrm{F}_{\mathrm{v}}{ }^{\prime}=\mathrm{F}_{\mathrm{v}}$ | x | $\mathrm{C}_{\mathrm{D}}$ | $\mathrm{C}_{\mathrm{M}}$ | $\mathrm{C}_{\text {t }}$ | - | - | - | $\mathrm{C}_{1}$ | - | - | - | - | 2.88 | 0.75 | $\lambda$ |
| $\mathrm{F}_{\mathrm{c}}{ }^{\prime}=\mathrm{F}_{\mathrm{c}}$ | x | $\underline{C_{D}}$ | $\mathrm{C}_{\mathrm{M}}$ | $\mathrm{C}_{\text {t }}$ | - | $\mathrm{C}_{\mathrm{F}}$ | - | $\mathrm{C}_{1}$ | - | $C_{P}$ | - | - | 2.40 | 0.90 | $\lambda$ |
| $\mathrm{F}_{\mathrm{c} \perp}{ }^{\prime}=\mathrm{F}_{\mathrm{c} \perp}$ | x | - | $\mathrm{C}_{\mathrm{M}}$ | $\mathrm{C}_{\text {t }}$ | - | - | - | $\mathrm{C}_{1}$ | - | - | - | $\mathrm{Cb}_{\mathrm{b}}$ | 1.67 | 0.90 | - |
| $E^{\prime}=E$ | x | - | $\mathrm{C}_{\mathrm{M}}$ | $\mathrm{C}_{\mathrm{t}}$ | - | - | - | $\mathrm{C}_{1}$ | - | - | - | - | - | - | - |
| $\mathrm{E}_{\text {min }}^{\prime}=\mathrm{E}_{\text {min }}$ | x | - | $\mathrm{C}_{\mathrm{M}}$ | $\mathrm{C}_{\text {t }}$ | - | - | - | $\mathrm{C}_{1}$ | - | - | $\mathrm{C}_{\mathrm{T}}$ | - | 1.76 | 0.85 | - |

## Allowable Flexure Stress $\mathrm{F}_{\mathrm{c}}{ }^{\text {' }}$

$F_{c}$ from tables determined by species and grade
$F_{c}^{\prime}=F_{c}\left(C_{D} C_{M} C_{t} C_{F} C_{i} C_{P}\right)$

Adjustment factors for compression:
$C_{D}$ Load Duration Factor
$\mathrm{C}_{\mathrm{t}}$ Temperature Factor

Table 2.3.3 Temperature Factor, $\mathbf{C}_{t}$

| Reference Design Values | $\begin{gathered} \text { In-Service } \\ \text { Moisture } \\ \text { Conditions }{ }^{1} \end{gathered}$ | $\mathrm{C}_{\text {t }}$ |  |  |
| :---: | :---: | :---: | :---: | :---: |
|  |  | $\mathrm{T} \leq 100^{\circ} \mathrm{F}$ | $100^{\circ} \mathrm{F}<\mathrm{T} \leq 125^{\circ} \mathrm{F}$ | $125^{\circ} \mathrm{F}<\mathrm{T} \leq 150^{\circ} \mathrm{F}$ |
| $\mathrm{F}_{\mathrm{t}}, \mathrm{E}, \mathrm{E}_{\text {min }}$ | Wet or Dry | 1.0 | 0.9 | 0.9 |
| $\mathrm{F}_{\mathrm{b}}, \mathrm{~F}_{\mathrm{v}}, \mathrm{~F}_{\mathrm{c}} \text {, and } \mathrm{F}_{\mathrm{c} \perp}$ | Dry | 1.0 | 0.8 | 0.7 |
|  | Wet | 1.0 | 0.7 | 0.5 |
| 1. Wet and dry service conditions for sawn lumber, structural glued laminated timber, prefabricated wood I-joists, structural composite lumber, wood structural panels and cross-laminated timber are specified in 4.1.4, 5.1.4, 7.1.4, 8.1.4, 9.3.3, and 10.1.5 respectively. |  |  |  |  |

Table 2.3.2 Frequently Used Load Duration Factors, $\mathbf{C D}^{1}{ }^{1}$

| Load Duration | $C_{D}$ | Typical Design Loads |
| :--- | :--- | ---: |
| Permanent | 0.9 | Dead Load |
| Ten years | 1.0 | Occupancy Live Load |
| Two months | 1.15 | Snow Load |
| Seven days | 1.25 | Construction Load |
| Ten minutes | 1.6 | Wind/Earthquake Load |
| Impact ${ }^{2}$ | 2.0 | Impact Load |

(1) Actual stress due to (DL) $\leq(0.9)$ (Design value)
(2) Actual stress due to (DL+LL)
(3) Actual stress due to ( $\mathrm{DL}+\mathrm{WL}$ )
(4) Actual stress due to (DL+LL+SL) $\leq(1.15)$ (Design value)
(5) Actual stress due to (DL+LL+WL) $\leq(1.6)$ (Design value)
(6) Actual stress due to (DL+SL+WL) $\leq(1.6)$ (Design value)
(7) Actual stress due to (DL+LL+SL+WL) $\leq(1.6)$ (Design value)

## Allowable Flexure Stress $\mathrm{F}_{\mathrm{c}}{ }^{\text {' }}$ (For Dimensioned Lumber)

$F_{c}$ from tables determined by species and grade
$F_{c}{ }^{\prime}=F_{c}\left(C_{D} C_{M} C_{t} C_{F} C_{i} C_{P}\right)$
Adjustment factors for compression:
$\mathrm{C}_{\mathrm{M}}$ Moisture Factor
$\mathrm{C}_{\mathrm{F}}$ Size Factor

Wet Service Factor, $\mathrm{C}_{\mathrm{M}}$
When dimension lumber is used where moisture content will exceed $19 \%$ for an extended time period, design values shall be multiplied by the appropriate wet service factors from the following table:

| Wet Service Factors, |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathrm{F}_{\mathrm{b}}$ | $\mathrm{F}_{\mathrm{t}}$ | $\mathrm{F}_{\mathrm{v}}$ | $\mathrm{F}_{\mathrm{c} \perp}$ | $\mathrm{F}_{\mathrm{c}}$ | $E$ and $\mathrm{E}_{\text {min }}$ |
| 0.85* | 1.0 | 0.97 | 0.67 | 0.8** | 0.9 |
| * when $\left(\mathrm{F}_{\mathrm{b}}\right)\left(\mathrm{C}_{\mathrm{F}}\right) \leq 1,150 \mathrm{psi}, \mathrm{C}_{\mathrm{M}}=1.0$ <br> ** when $\left(\mathrm{F}_{\mathrm{c}}\right)\left(\mathrm{C}_{\mathrm{F}}\right) \leq 750$ psi, $\mathrm{C}_{\mathrm{M}}=1.0$ |  |  |  |  |  |


| Size Factorr $\widehat{\mathrm{C}_{\mathrm{F}}}$ |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Grades | Width (depth) | $\mathrm{F}_{\mathrm{b}}$ |  | $\mathrm{F}_{\mathrm{t}}$ | $\underline{F_{c}}$ |
|  |  | Thickness (breadth) |  |  |  |
|  |  | $2^{\prime \prime}$ \& 3" | 4" |  |  |
| Select <br> Structural, <br> No. 1 \& Btr, <br> No.1, No.2, <br> No. 3 | 2", $3^{\prime \prime}$, \& 4" | 1.5 | 1.5 | 1.5 | 1.15 |
|  | 5 | 1.4 | 1.4 | 1.4 | 1.1 |
|  | $6 "$ | 1.3 | 1.3 | 1.3 | 1.1 |
|  | 8" | 1.2 | 1.3 | 1.2 | 1.05 |
|  | $10^{\prime \prime}$ | 1.1 | 1.2 | 1.1 | 1.0 |
|  | 12 " | 1.0 | 1.1 | 1.0 | 1.0 |
|  | 14 " \& wider | 0.9 | 1.0 | 0.9 | 0.9 |
| Stud | 2", 3", \& 4" | 1.1 | 1.1 | 1.1 | 1.05 |
|  | $5 " \& 6 "$ | 1.0 | 1.0 | 1.0 | 1.0 |
|  | 8" \& wider | Use No. 3 Grade tabulated design values and size fac |  |  |  |
| Construction, Standard | $2^{\prime \prime}, 3^{\prime \prime}, \& 4^{\prime \prime}$ | 1.0 | 1.0 | 1.0 | 1.0 |
| Utility | 4" | 1.0 | 1.0 | 1.0 | 1.0 |
|  | 2" \& 3" | 0.4 | - | 0.4 | 0.6 |

## Allowable Flexure Stress $\mathrm{F}_{\mathrm{c}}{ }^{\text {' }}$ (For Timbers)

$F_{c}$ from tables determined by species and grade
$F_{c}{ }^{\prime}=F_{c}\left(C_{D} C_{M} C_{t} C_{F} C_{i} C_{P}\right)$
Adjustment factors for compression:
$\mathrm{C}_{\mathrm{M}}$ Moisture Factor
$\mathrm{C}_{\mathrm{F}}$ Size Factor

## Size Factor, $\mathrm{C}_{\mathrm{F}}$

When visually graded timbers are subjected to loads applied to the narrow face, tabulated design values shall be multiplied by the following size factors:


## Wet Service Factor, $\mathrm{C}_{\mathrm{M}}$

When timbers are used where moisture content will exceed $19 \%$ for an extended time period, design values shall be multiplied by the appropriate wet service factors from the following table (for Southern Pine and Mixed Southern Pine, use tabulated design values without further adjustment):

Wet Service Factors, $\mathrm{C}_{\mathrm{m}}$

| $\mathrm{F}_{\mathrm{b}}$ | $\mathrm{F}_{\mathrm{t}}$ | $\mathrm{F}_{\mathrm{v}}$ | $\mathrm{F}_{\mathrm{c} \perp}$ | $\mathrm{F}_{\mathrm{c}}$ | E and $\mathrm{E}_{\text {min }}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1.00 | 1.00 | 1.00 | 0.67 | 0.91 | 1.00 |

## Allowable Flexure Stress $\mathrm{F}_{\mathrm{c}}{ }^{\prime}$

$F_{c}$ from tables determined by species and grade
$F_{c}{ }^{\prime}=F_{c}\left(C_{D} C_{M} C_{t} C_{F} C_{i} C_{P}\right)$

Adjustment factors for compression :
$\mathrm{C}_{\mathrm{i}}$ Incising Factor

Table 4.3.8 Incising Factors, $\mathbf{C}_{\mathbf{i}}$

| Design Value | $\mathbf{C}_{\mathbf{i}}$ |
| :--- | :--- |
| $\mathrm{E}, \mathrm{E}_{\min }$ | 0.95 |
| $\mathrm{~F}_{\mathrm{b}}, \mathrm{F}_{\mathrm{t}} \mathrm{F}_{\mathrm{c}} \mathrm{F}_{\mathrm{v}}$ | 0.80 |
| $\mathrm{~F}_{\mathrm{c} \perp}$ | 1.00 |



## Allowable Flexure Stress $\mathrm{F}_{\mathrm{c}}{ }^{\text {' }}$

$F_{c}$ from tables determined by species and grade

$$
F_{c}^{\prime}=F_{c}\left(C_{D} C_{M} C_{t} C_{F} C_{i} C_{P}\right)
$$

### 3.7 Solid Columns

### 3.7.1 Column Stability Factor, $\mathbf{C}_{\boldsymbol{p}}$


3.7.1.1 When a compression member is supported throughout its length to prevent lateral displacement in all directions, $\mathrm{C}_{\mathrm{P}}=1.0$.
3.7.1.2 The effective column length, $\ell_{e}$, for a solid column shall be determined in accordance with principles of engineering mechanics. One method for determining effective column length, when end-fixity conditions are known, is to multiply actual column length by the appropriate effective length factor specified in Appendix G, $\ell_{\mathrm{e}}=\left(\mathrm{K}_{\mathrm{e}}\right)(\boldsymbol{\ell})$.
3.7.1.3 For solid columns with rectangular cross section, the slenderness ratio, $\ell_{\mathrm{e}} / \mathrm{d}$, shall be taken as the larger of the ratios $\ell_{\mathrm{e} 1} / \mathrm{d}_{1}$ or $\ell_{\mathrm{e} 2} / \mathrm{d}_{2}$ (see Figure 3 F ) where each ratio has been adjusted by the appropriate buckling length coefficient, $\mathrm{K}_{\mathrm{e}}$, from Appendix G .
3.7.1.4 The slenderness ratio for solid columns, $\ell_{d} / \mathrm{d}$, shall not exceed 50 except that during construction $\ell_{\mathrm{e}} / \mathrm{d}$ shall not exceed 75 .
3.7.1.5 The column stability factor shall be calculated as follows:
$C_{P}=\frac{1+\left(F_{O E} / F_{\mathrm{c}}^{*}\right)}{2 \underline{c}}-\sqrt{\left[\frac{1+\left(\mathrm{F}_{\mathrm{OE}} / \mathrm{F}_{\mathrm{C}}^{*}\right)}{2 \mathrm{c}}\right]^{2}-\frac{\mathrm{F}_{\mathrm{CE}} / \mathrm{F}_{\mathrm{c}}^{*}}{\mathrm{c}}}$
where:
$\mathrm{F}_{\mathrm{c}}{ }^{*}=$ reference compression design value parallel to grain multiplied by all applicable adjustment factors except $\mathrm{C}_{\mathrm{p}}$ (see 2.3), psi
$\mathrm{F}_{\mathrm{CE}}=\frac{0.822 \mathrm{E}_{\text {min }}{ }^{\prime}}{\left(\ell_{\mathrm{e}} / \mathrm{d}\right)^{2}}$
$c=0.8$ for sawn lumber
$\mathrm{c}=0.85$ for round timber poles and piles
$c=0.9$ for structural glued laminated timber or structural composite lumber

## $C_{P}$ estimation



## Analysis of Wood Columns

## Data:

- Column - size, length
- Support conditions
- Material properties - $\mathrm{F}_{\mathrm{c}}$, E
- Load


## Required:

- Pass/Fail or margin of safety

1. Calculate slenderness ratio $\ell_{\mathrm{e}} / \mathrm{d}$ largest ratio governs. Must be $<50$ -

2. Find adjustment factors (all except $\mathrm{C}_{\mathrm{P}}$ ) $C_{D} C_{M} C_{t} C_{F}$
3. Calculate $\mathrm{C}_{\mathrm{P}}$
4. Determine F'c by multiplying the tabulated Fc by all the above factors
5. Calculate the actual stress: $\mathrm{fc}=P / A$
6. Compare Allowable and Actual stress.

F'c >fc passes

## Analysis Example:

Data: section $4 \times 8$ (nominal) Douglas Fir-Larch No M.C. 15\% P = 7000 LBS (Snow Load)

Find: Pass/Fail


From NDS Supplement Table 4A
Pc $=1500 \mathrm{psi}$ Amin $=620000$ psi
$C_{D}=1.15$ (snow)
$\mathrm{C}_{\mathrm{M}}=1.0$
$\mathrm{C}_{\mathrm{t}}=1.0$ -
$C_{F}=1.05(4 \times 8)$
$\mathrm{C}_{\mathrm{i}}=1.0$ -
$C_{P}=$ ?
$\qquad$

## Analysis Example:

Calculate $\mathrm{C}_{\mathrm{P}}$

$$
\begin{equation*}
C_{p}=\frac{1+\left(F_{c E} / F_{\mathrm{c}}^{*}\right)}{2 \mathrm{c}}-\sqrt{\left[\frac{1+\left(\mathrm{F}_{\mathrm{cE}} / \mathrm{F}_{\mathrm{c}}^{*}\right)}{2 \mathrm{c}}\right]^{2}-\frac{\mathrm{F}_{\mathrm{cE}} / \mathrm{F}_{\mathrm{c}}^{*}}{\mathrm{c}}} \tag{3.7-1}
\end{equation*}
$$

where:
$\mathrm{F}_{\mathrm{c}}{ }^{\circ}=$ reference compression design value aralleI to grain multiplied by all applicable adjustment factors except $\mathrm{C}_{\mathrm{p}}$ (see 2.3), psi
$\mathrm{F}_{\mathrm{cE}}=\frac{0.822 \mathrm{E}_{\mathrm{mn}}{ }^{\prime}}{\left(\ell_{\mathrm{e}} / \mathrm{d}\right)^{2}}$
$c=0.8$ for saw lumber
$\mathrm{c}=0.85$ for round timber poles and piles
$\mathrm{c}=0.9$ for structural glued laminated timber or structural composite lumber


$$
\begin{array}{rlrl}
x-x & y-y \\
l_{x}=25^{\prime}=\frac{300^{\prime \prime}}{} & \operatorname{le}_{y}=10^{\prime} & =120^{\prime \prime} \\
l_{e x} / d_{1}=\frac{300^{\prime \prime}}{7.25^{\prime \prime}} & l_{e y} / d_{2} & =\frac{120^{\prime \prime}}{3.5^{\prime \prime}} \\
& =41.4 & & =34.3 \\
l e l d & =41.4<50
\end{array}
$$

## Analysis Example:

Calculate $\mathrm{C}_{\mathrm{P}}$

$$
\begin{equation*}
\mathrm{C}_{\mathrm{p}}=\frac{1+\left(\mathrm{F}_{\mathrm{cE}} / \mathrm{F}_{\mathrm{c}}^{*}\right)}{2 \mathrm{c}}-\sqrt{\left[\frac{1+\left(\mathrm{F}_{\mathrm{cE}} / \mathrm{F}_{\mathrm{c}}^{*}\right)}{2 \mathrm{c}}\right]^{2}-\frac{\mathrm{F}_{\mathrm{cE}} / \mathrm{F}_{\mathrm{c}}^{*}}{\underline{\mathrm{c}}}} \tag{3.7-1}
\end{equation*}
$$

where:
$F_{c}{ }^{*}=$ reference compression design value paraltel to grain multiplied by all applicable adjustment factors except $\mathrm{C}_{\mathrm{p}}$ (see 2.3), psi
$\mathrm{F}_{\mathrm{cE}}=\frac{0.822 \mathrm{E}_{\text {min }}{ }^{\prime}}{\left(\ell_{\mathrm{e}} / \mathrm{d}\right)^{2}}$
$c=0.8$ for saw lumber
$c=0.85$ for round timber poles and piles
$c=0.9$ for structural glued laminated timber or structural composite lumber

$$
\begin{aligned}
F_{\text {CE }} & =\frac{0.822 E_{\text {min }}^{\prime}}{\left(\mathrm{le}_{e} / \mathrm{d}\right)^{2}} \\
& =\frac{0.822(620000)}{(41.4)^{2}} \\
& =297.6 \mathrm{psi}- \\
F_{C}^{*} & =1500(1.15 \mathrm{p} .05) \\
& =\frac{1811.25 \mathrm{pi}}{\mathrm{pi}} \\
F_{C E} / F_{C}^{*} & =\frac{297.6}{1811.25}=0.164 \\
C & =0.8
\end{aligned}
$$

## Analysis Example:

Calculate $\mathrm{C}_{\mathrm{P}}$
$C_{p}=\frac{1+\left(F_{\mathrm{c}} / F_{\mathrm{c}}^{*}\right)}{2 \mathrm{c}}-\sqrt{\left[\frac{1+\left(\mathrm{F}_{\mathrm{cE}} / F_{\mathrm{c}}^{*}\right)}{2 \mathrm{c}}\right]^{2}-\frac{\mathrm{F}_{\mathrm{cE}} / \mathrm{F}_{\mathrm{c}}}{\mathrm{c}}}$
where:
$\mathrm{F}_{\mathrm{c}}{ }^{\circ}=$ reference compression design value paralleI to grain multiplied by all applicable adjustment factors except $\mathrm{C}_{\mathrm{p}}$ (see 2.3), psi
$F_{c E}=\frac{0.822 E_{\text {min }}{ }^{\prime}}{\left(\ell_{\mathrm{e}} / \mathrm{d}\right)^{2}}$
$\mathrm{c}=0.8$ for awn lumber
$c=0.85$ for round timber poles and piles
$\mathrm{c}=0.9$ for structural glued laminated timber or structural composite lumber

Compare Allowable and Actual stress F'c >fo passes

$$
C_{p}=\frac{1+0.164}{2(0.8)}-\sqrt{\left[\frac{1+0.164}{2(0.8)}\right]^{2}-\frac{0.164}{.8}}
$$

$$
C_{p}=0.1584
$$

$$
\begin{aligned}
C_{D} & C_{F} \\
F_{C}^{\prime} & =1500(1.15 \\
& C_{p} \\
& \left.=\frac{286.9}{} 0.1584\right) \\
f_{C} & =\frac{P}{A}=\frac{7000^{*}}{25.38 \mathrm{~m}^{2}}=275.8 \mathrm{ps} 1
\end{aligned}
$$

$\mathrm{F}_{\mathrm{c}}^{\prime}>\mathrm{Fe}_{\mathrm{c}} \checkmark$ ok $286>275$

## Capacity Analysis of Columns

## Data:

- Column - size, length
- Support conditions
- Material properties $-\mathrm{F}_{\mathrm{c}}, \mathrm{E}$


## Required:

- Maximum Load Pmax

1. Calculate slenderness ratio $\ell_{\mathrm{e}} / \mathrm{d}$ largest ratio governs. Must be < 50
2. Find adjustment factors (all except $\mathrm{C}_{\mathrm{P}}$ )

$$
C_{D} C_{M} C_{t} C_{F} C_{i}
$$


3. Calculate $\mathrm{C}_{\mathrm{P}}$ -
4. Determine F'c by multiplying the tabulated Fc by all the above factors
5. Set actual stress $=$ allowable, $\mathrm{fc}=\mathrm{F}$ ' C
6. Find the maximum allowable load

$$
\text { Pmax }=\text { F'c A }
$$

## Capacity Example

## Data:

- $4 \times 10$
- Hem - Fir, No 2 M.C. $=\underline{20 \%}$
- Wind Load
- $\mathrm{L}_{1}=8^{\prime} \mathrm{L}_{2}=4^{\prime} \mathrm{K}_{\mathrm{e}}=1.0$


## Required:

- Maximum Load, Pmax

From NDS Supplement Table 4A
Fc = 1300 psi


Emin $=470000$ psi
$C_{D}=1.6$ WIND
$C_{M C}=0.8 \quad C_{M E}=0.9$
$C_{t}=1.0$
$C_{F}=1.0$
$C_{i}=1.0$
$C_{P}=? \approx 0.75$

## Allowable Flexure Stress $\mathrm{F}_{\mathrm{c}}{ }^{\text {' }}$

$4 \times 10$ M.C $20 \%$ Kc $=\underline{1300}$ psi
$F_{c}$ from tables determined by species and grade
$F_{c}{ }^{\prime}=F_{c}\left(C_{D} C_{M} C_{t} C_{F} C_{i} C_{P}\right)$
Adjustment factors for compression:
$\mathrm{C}_{\mathrm{M}}$ Moisture Factor
$\mathrm{C}_{\mathrm{F}}$ Size Factor

Wet Service Factor, $\mathrm{C}_{\mathrm{M}}$
When dimension lumber is used where moisture content will exceed $19 \%$ for an extended time period, design values shall be multiplied by the appropriate wet service factors from the following table:



## Capacity Example

Find $C_{P}$

$$
\begin{aligned}
F_{C E} & =\frac{0.822 E_{\text {min }}^{\prime}}{\left(l_{e} / d\right)^{2}} \\
& =\frac{0.822(470000(0.9))}{13.7^{2}} \\
& =1848.7 \mathrm{psi}
\end{aligned}
$$

$$
\begin{aligned}
F_{c}^{*} & =1300(1.60 .8) \\
& =1664 \cdot \mathrm{psi}
\end{aligned}
$$

$$
F_{C E} / \xi_{C}^{*}=\frac{1848.7}{1664}=1.111
$$

$$
C_{p}=0.7261
$$

$$
\begin{equation*}
C_{p}=\frac{1+\left(\mathrm{F}_{\mathrm{cE}} / \mathrm{F}_{\mathrm{c}}^{*}\right)}{2 \mathrm{c}}-\sqrt{\left[\frac{1+\left(\mathrm{F}_{\mathrm{cE}} / \mathrm{F}_{\mathrm{c}}^{*}\right)}{2 \mathrm{c}}\right]^{2}-\frac{\mathrm{F}_{\mathrm{cE}} / \mathrm{F}_{\mathrm{c}}^{*}}{\mathrm{c}}} \tag{3.7-1}
\end{equation*}
$$

where:
$\mathrm{F}_{\mathrm{c}}{ }^{*}=$ reference compression design value aralleI to grain multiplied by all applicable adjustment factors except $\mathrm{C}_{\mathrm{p}}$ (see 2.3), psi
$\mathrm{F}_{\mathrm{cE}}=\frac{0.822 \mathrm{E}_{\text {min }}^{\prime}}{\left(\ell_{\mathrm{e}} / \mathrm{d}\right)^{2}}$
$c=0.8$ for saw lumber
$c=0.85$ for round timber poles and piles
$c=0.9$ for structural glued laminated timber or structural composite lumber

Find the maximum load, Pax

$$
\begin{aligned}
& =1208 \mathrm{p}^{5} \\
& P_{\text {max }}=F_{c}^{\prime} A=1208(32.38)=39115^{*}
\end{aligned}
$$

## Timber Column Design

Given:

- Lumber species, grade
- Conditions of use $C_{p}-C_{F}$
- Load


## Required:

- column size

1. Find adjustment factors (all except $\mathrm{C}_{\mathrm{P}}$ )

$$
C_{D} C_{M} C_{t} C_{F} C_{i}
$$

$\rightarrow$ 2. Guess $C_{P}$
3. Estimate Area and d (based on bracing)
4. Calculate slenderness ratio $I_{e} / d$ largest ratio governs. Must be < 50
5. Calculate $\mathrm{C}_{\mathrm{P}}$
6. Determine F'c by multiplying the tabulated Fc by all the above factors
7. Revise Area: $A=P / F ' c$

8. Revise $C_{p}$

## Timber Column Design

## Given:

- White Oak, No. 1
- dry use, normal temp., not incised
- Load: D+L=55 psf


## Required:

- column size

1. Find adjustment factors (all except $\mathrm{C}_{\mathrm{P}}$ )
$C_{D} C_{M} C_{t} \dot{C}_{F} C_{i}$

2. Guess $\underline{C}_{P} \rightarrow$ try 0.5

Table 4D Reference Design Values for Visually Graded Timbers (5" x 5" and larger) ${ }^{1,3}$
(Cont.) (Tabulated design values are for normal load duration and dry service conditions, unless specified otherwise. See NDS 4.3 for a comprehensive description of design value adjustment factors.)

| USE WITH TABLE 4D ADJUSTMENT FACTORS |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Species and commercial Grade | Size classification | Design values in pounds per square inch (psi) |  |  |  |  |  |  | Specific Gravity ${ }^{4}$ G | Grading Rules Agency |
|  |  | $\begin{gathered} \text { Bending } \\ F_{b} \\ \hline \end{gathered}$ | Tension parallel to grain$\qquad$$\mathrm{F}_{\mathrm{t}}$ | Shear parallel to grain$F_{v}$ | Compression perpendicular to grain <br> $\mathrm{F}_{\mathrm{c} \perp}$ | Compression parallel to grain $\mathrm{F}_{\mathrm{c}}$ | Modulus of Elasticity |  |  |  |
|  |  |  |  |  |  |  | E | $\mathrm{E}_{\text {min }}$ |  |  |
| WHITE OAK |  |  |  |  |  |  |  |  |  |  |
| Select Structural | Beams and Stringers | $\begin{aligned} & 1,400 \\ & 1,200 \\ & 750 \end{aligned}$ | $\begin{aligned} & 825 \\ & 575 \\ & 375 \end{aligned}$ | $\begin{aligned} & 205 \\ & 205 \\ & 205 \end{aligned}$ | 800 <br> 800 <br> 800 | 900775475 | $\begin{gathered} \hline 1,000,000 \\ 1,000,000 \\ 800,000 \\ \hline \end{gathered}$ | $\begin{aligned} & \hline 370,000 \\ & 370,000 \\ & 290,000 \\ & \hline \end{aligned}$ | 0.73 | NELMA |
| No. 1 |  |  |  |  |  |  |  |  |  |  |
| No. 2 |  |  |  |  |  |  |  |  |  |  |
| Select Structural | Posts and | 1,300 | 875 | 205 | 800 | 950 | 1,000,000 | 370,000 |  |  |
| No. 1 |  | 1,050 | 700 | 205 | 800 | 825 | 1,000,000 | 370,000 |  |  |
| No. 2 |  | 600 | 400 | 205 | 800 | 400 | 800,000 | 290,000 |  |  |

## Timber Column Design

## Given:

- White Oak, No. 1
- dry use, normal temp., not incised
- Load: D+L=55 psf

Required:

- column size


1. Find adjustment factors (all except $\mathrm{C}_{\mathrm{P}}$ )

$$
C_{D} C_{M} C_{t} C_{F} C_{i}
$$

2. Guess $C_{P} \rightarrow$ try 0.5
3. Estimate Area and d (based on bracing)
4. Calculate slenderness ratio $I_{e} / d$ largest ratio governs. Must be < 50

$$
\begin{aligned}
& \text { ESTIMATE SIZE: } \\
& \text { CURS } C_{p}=0.5 \\
& A=\frac{p}{F_{C}^{\prime}}=\frac{14080 *^{\prime}}{825(.5)}=34 \mathrm{~m}^{2} \\
& T_{R} Y: \\
& \sqrt{A}=d \quad \sqrt{34^{\prime}}=5.8^{\prime \prime} \\
& \text { SAY } 5.5^{\prime \prime} \times 5.5^{\prime \prime}
\end{aligned}
$$

## Timber Column Design

## Given:

- White Oak, No. 1

TRy $6 \times 6$

- dry use, normal temp., not incised
- Load: D+L=55 psf

Required:

$$
\frac{l_{e}}{d}=\frac{(1) 144^{\prime \prime}}{5.5}=26.18
$$

- column size

$$
\text { CHECK } C_{p} \text { on LRAIH - SDJOST IF NEEDED }
$$

$$
F_{C_{E}}=\frac{0.822(370000)}{26.18^{2} 1.0}=443.7 \mathrm{psi}
$$

5. Calculate $\mathrm{C}_{\mathrm{P}}$

$$
F_{C}^{*}=F_{C}\left(C_{D} C_{M} C_{F} C_{t} C_{i}\right)=825 \mathrm{PSI}
$$

$$
F_{C E} / F_{c}^{*}=\frac{443.7}{E 25}=0.5378
$$



## Timber Column Design

Given:

- White Oak, No. 1
- dry use, normal temp., not incised
- Load: D+L=55 psf


## Required:

- column size

6. Determine F'c by multiplying the tabulated Ec by all the above factors
7. Revise Area: A = P/F'c
8. Revise $C_{P}$

Table 1B Section Properties of Standard Dressed


Timbers (5" $\times 5$ 5" and larger) ${ }^{2}$
Post and Timber (see NDS 4.1.3.4 and NDS 4.1.5.3)

| $5 \times 5$ | $4-1 / 2 \times 4-1 / 2$ | 20.25 | 15.19 | 34.17 | 15.19 | 34.17 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $6 \times 6$ | $5-1 / 2 \times 5-1 / 2$ | 30.85 | 27.73 | 76.26 | 27.73 | 76.26 |
| $6 \times 8$ | $5-1 / 2 \times 7-1 / 2$ | 41.25 | 51.56 | 193.4 | 37.81 | 104.0 |
| $8 \times 8$ | $7-1 / 2 \times 7-1 / 2$ | 56.25 | 70.31 | 263.7 | 70.31 | 263.7 |
| $8 \times 10$ | $7-1 / 2 \times 9-1 / 2$ | 71.25 | 112.8 | 535.9 | 89.06 | 334.0 |

$$
\begin{aligned}
& \text { REVISED } \frac{F_{c}^{\prime}}{C_{p}} \\
& F_{C}^{\prime}=825(0.46)=379.5 \\
& A=\frac{P}{F_{C}^{\prime}}=\frac{14080}{379.5 \%^{2} / m^{2}}=37.1 \mathrm{~m}^{2} \\
& 6 \times 6: A=30.25<37.1 \therefore F A 1 L S \\
& 6 \times 8=41.25 \mathrm{~m}^{2}>37.1
\end{aligned}
$$

$$
\text { TRy } 6 \times 8
$$

$$
l_{e} / d=\frac{144^{\prime \prime}}{5.5^{\prime \prime}}=\frac{26.18}{\left(\sin 4 \theta^{2} 6 \times 6\right)}
$$

$$
c_{p}=0.46 \text { (No cis, }
$$

$$
\therefore 6 \times 8 \text { PAsses }
$$

## Timber Column Design

## Design Aids

ASD/LRFD MANUAL FOR ENGINEERED WOOD CONSTRUCTION
example of a column chart
from AWC Manual for Engineered
Wood Construction - 2005
14080 *


