Wood Columns

- Failure Modes
- Euler Equation
- End Conditions and Lateral Bracing
- Analysis of Wood Columns
- Design of Wood Columns

Failure Modes

**FLEXURE**

- Strength
  \[
  f_b = \frac{Mc}{I}, \quad f_v = \frac{VQ}{Ib}
  \]

- Stability
  \[
  C_L = \frac{1 + \left( \frac{F_{le}}{F_b^*} \right)}{1.9} - \sqrt{\left[ \frac{1 + \left( \frac{F_{le}}{F_b^*} \right)}{1.9} \right]^2 - \frac{F_{le}}{F_b^*}}
  \]

**AXIAL**

- Strength
  \[
  f_c = \frac{P}{A}
  \]

- Serviceability
  **Deflection**
  **Bearing (crushing limit)**
Leonhard Euler (1707 – 1783)

Euler Buckling (elastic buckling)

\[ P_{cr} = \frac{\pi^2 AE}{(KL/r)^2} = \frac{\pi^2 IE}{KL^2} \]

- \( A \) = Cross sectional area (in\(^2\))
- \( E \) = Modulus of elasticity of the material (lb/in\(^2\))
- \( K \) = Stiffness (curvature mode) factor
- \( L \) = Column length between pinned ends (in.)
- \( r \) = radius of gyration (in.)

\[ r = \sqrt{\frac{I}{A}} \]

\[ I = Ar^2 \]

Failure Mode - Strength

**Short Columns** – fail by crushing

\[ f_c = \frac{P}{A} \leq F_c \]

- \( f_c \) = Actual compressive stress
- \( A \) = Cross-sectional area of column (in\(^2\))
- \( P \) = Load on the column
- \( F_c \) = Allowable compressive stress per codes
Failure Modes – Stability

Long Columns – fail by buckling

Traditional Euler

\[ f_{cr} = \frac{\pi^2 E}{(KL/r)^2} \]

- \( E \) = Modulus of elasticity of the column material (psi)
- \( K \) = Stiffness (curvature mode) factor
- \( L \) = Column length between ends (inches)
- \( r \) = radius of gyration = \((I/A)^2\) (inches)

NDS Equation

\[ F_{cE} = \frac{0.822E_{min}'}{(l_e/d)^2} \]

- \( E_{\text{min}'} \) = reduced \( E \) modulus (psi)
- \( l_e = Ke \) lu
- \( d \) (inches)
- \( 0.822 = \pi^2/12 \)

\[ r = d/\sqrt{12} \]

Slenderness Ratio

Slenderness Ratios:

The larger ratio will govern.
Try to balance for efficiency.

Slenderness Limit to < 50

\[ \frac{L_e}{d} = \frac{96}{3.5} = 27.4 \]

\[ \frac{L_e}{b} = \frac{96}{1.5} = 64 \]
End Support Conditions

\( K_e \) is a constant based on the end conditions

\( l \) is the actual length

\( l_e \) is the effective length (curved part)

\[ l_e = K_e l \]

### Table G1

<table>
<thead>
<tr>
<th>Buckling mode</th>
<th>Theoretical ( K_e ) value</th>
<th>Recommended design ( K_e ) when ideal conditions approximated</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.5</td>
<td>0.66</td>
</tr>
<tr>
<td></td>
<td>0.7</td>
<td>0.80</td>
</tr>
<tr>
<td></td>
<td>1.0</td>
<td>1.2</td>
</tr>
<tr>
<td></td>
<td>2.0</td>
<td>1.0</td>
</tr>
<tr>
<td></td>
<td>2.0</td>
<td>2.10</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2.4</td>
</tr>
</tbody>
</table>

**End condition code**
- Rotation fixed, translation fixed
- Rotation free, translation fixed
- Rotation fixed, translation free
- Rotation free, translation free

Use these

---

**Allowable Flexure Stress** \( F_c' \)

\( F_c \) from tables determined by species and grade

\( F_c' = F_c \) (adjustment factors)

\[ F_c' \geq f_c \]

**Actual Flexure Stress** \( f_b \)

\[ f_c = \frac{P}{A} \]

---

**Table 4A** Base Design Values for Visually Graded Dimension Lumber (2"-4" thick)\(^{1,2}\)

(All species except Southern Pine — see Table 4B) (Tabulated design values are for normal load duration and dry service conditions. See NDS 4.3 for a comprehensive description of design value adjustment factors.)
Adjustment Factors

### Table 4.3.1 Applicability of Adjustment Factors for Sawn Lumber

<table>
<thead>
<tr>
<th>Adjustment Factors</th>
<th>ASD only</th>
<th>ASD and LRFD</th>
<th>LRFD only</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Load Duration Factor</td>
<td>Wet Service Factor</td>
<td>Temperature Factor</td>
</tr>
<tr>
<td>$F_{c}^{'} = F_{b}$</td>
<td>X</td>
<td>C_D</td>
<td>C_M</td>
</tr>
<tr>
<td>$F_{c}^{'} = F_{l}$</td>
<td>X</td>
<td>C_D</td>
<td>C_M</td>
</tr>
<tr>
<td>$F_{c}^{'} = F_{v}$</td>
<td>X</td>
<td>C_D</td>
<td>C_M</td>
</tr>
<tr>
<td>$F_{c}^{'} = F_{z}$</td>
<td>X</td>
<td>C_D</td>
<td>C_M</td>
</tr>
<tr>
<td>$F_{c,\ell}^{'} = F_{c,\ell}$</td>
<td>X</td>
<td>-</td>
<td>C_M</td>
</tr>
<tr>
<td>$E^{'} = E$</td>
<td>X</td>
<td>-</td>
<td>C_M</td>
</tr>
<tr>
<td>$E_{\min}^{'} = E_{\min}$</td>
<td>X</td>
<td>-</td>
<td>C_M</td>
</tr>
</tbody>
</table>

**Adjustment Factors for Compression:**
- $C_D$: Load Duration Factor
- $C_M$: Temperature Factor

**Allowable Flexure Stress $F_{c}^{'}$**

$F_{c}^{'} = F_{c} \left( C_D \times C_M \times C_F \times C_t \right)$

Adjustment factors for compression:
- $C_D$: Load Duration Factor
- $C_t$: Temperature Factor

### Table 2.3.2 Frequently Used Load Duration Factors, $C_{D}$

<table>
<thead>
<tr>
<th>Load Duration</th>
<th>$C_D$</th>
<th>Typical Design Loads</th>
</tr>
</thead>
<tbody>
<tr>
<td>Permanent</td>
<td>0.9</td>
<td>Dead Load</td>
</tr>
<tr>
<td>Ten years</td>
<td>1.0</td>
<td>Occupancy Live Load</td>
</tr>
<tr>
<td>Two months</td>
<td>1.15</td>
<td>Snow Load</td>
</tr>
<tr>
<td>Seven days</td>
<td>1.25</td>
<td>Construction Load</td>
</tr>
<tr>
<td>Ten minutes</td>
<td>1.6</td>
<td>Wind/Earthquake Load</td>
</tr>
<tr>
<td>Impact2</td>
<td>2.0</td>
<td>Impact Load</td>
</tr>
</tbody>
</table>

1. Actual stress due to (DL) ≤ (0.9) (Design value)
2. Actual stress due to (DL+LL) ≤ (1.0) (Design value)
3. Actual stress due to (DL+WL) ≤ (1.6) (Design value)
4. Actual stress due to (DL+LL+SL) ≤ (1.15) (Design value)
5. Actual stress due to (DL+LL+WL) ≤ (1.6) (Design value)
6. Actual stress due to (DL+SL+WL) ≤ (1.6) (Design value)
7. Actual stress due to (DL+LL+SL+WL) ≤ (1.6) (Design value)

### Table 2.3.3 Temperature Factor, $C_{t}$

<table>
<thead>
<tr>
<th>Reference Design Values</th>
<th>In-Service Moisture Conditions1</th>
<th>$C_t$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$TS_{100^\circ}F$</td>
<td>$TS_{125^\circ}F$</td>
</tr>
<tr>
<td>$F_{w}$, $E_{\min}$</td>
<td>Wet and Dry</td>
<td>1.0</td>
</tr>
<tr>
<td>$F_{w}$, $F_{c}$, and $F_{c,\ell}$</td>
<td>Wet and Dry</td>
<td>1.0</td>
</tr>
</tbody>
</table>

1. Wet and dry service conditions for lumber, structural glued laminated timber, prefinished wood L-products, structural composite lumber, wood structural panels and cross-laminated timber are specified at 4.3.4, 5.1.4, 7.1.4, 8.1.4, 9.3.3, and 10.1.3 respectively.

- (1) Actual stress due to (DL) ≤ (0.9) (Design value)
- (2) Actual stress due to (DL+LL) ≤ (1.0) (Design value)
- (3) Actual stress due to (DL+WL) ≤ (1.6) (Design value)
- (4) Actual stress due to (DL+LL+SL) ≤ (1.15) (Design value)
- (5) Actual stress due to (DL+LL+WL) ≤ (1.6) (Design value)
- (6) Actual stress due to (DL+SL+WL) ≤ (1.6) (Design value)
- (7) Actual stress due to (DL+LL+SL+WL) ≤ (1.6) (Design value)
Allowable Flexure Stress $F_{c'}$
(For Dimensioned Lumber)

$F_c$ from tables determined by species and grade

$F_{c'} = F_c \cdot (C_D \cdot C_M \cdot C_t \cdot C_F \cdot C_i \cdot C_p)$

Adjustment factors for compression:
- $C_M$ Moisture Factor
- $C_F$ Size Factor

---

Allowable Flexure Stress $F_{c'}$
(For Timbers)

$F_c$ from tables determined by species and grade

$F_{c'} = F_c \cdot (C_D \cdot C_M \cdot C_t \cdot C_F \cdot C_i \cdot C_p)$

Adjustment factors for compression:
- $C_M$ Moisture Factor
- $C_F$ Size Factor
Allowable Flexure Stress $F_{c'}$

$F_{c'}$ from tables determined by species and grade

$F_{c'} = F_c (C_D \ C_M \ C_t \ C_F \ C_i \ C_P )$

Adjustment factors for compression:

$C_i$ Incising Factor

### Table 4.3.8 Incising Factors, $C_i$

<table>
<thead>
<tr>
<th>Design Value</th>
<th>$C_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E, E_{min}$</td>
<td>0.95</td>
</tr>
<tr>
<td>$F_b, F_o, F_v$</td>
<td>0.80</td>
</tr>
<tr>
<td>$F_{ci}$</td>
<td>1.00</td>
</tr>
</tbody>
</table>

3.7 Solid Columns

#### 3.7.1 Column Stability Factor, $C_p$

3.7.1.1 When a compression member is supported throughout its length to prevent lateral displacement in all directions, $C_p = 1.0$.

3.7.1.2 The effective column length, $\ell_e$, for a solid column shall be determined in accordance with principles of engineering mechanics. One method for determining effective column length, when end-fixity conditions are known, is to multiply actual column length by the appropriate effective length factor specified in Appendix G, $\ell_e = (K_e)\ell$.

3.7.1.3 For solid columns with rectangular cross section, the slenderness ratio, $\ell_e/d$, shall be taken as the larger of the ratios $\ell_{crit}/d$, or $\ell_{crit}/d_2$ (see Figure 3F) where each ratio has been adjusted by the appropriate buckling length coefficient, $K_{non}$ from Appendix G.

3.7.1.4 The slenderness ratio for solid columns, $\ell_{crit}/d$, shall not exceed 50 except that during construction $\ell_{crit}/d$ shall not exceed 75.

3.7.1.5 The column stability factor shall be calculated as follows:

$$C_p = \frac{1 + \left( \frac{F_{ci} / F_c'}{2c} \right)}{2c} - \sqrt{\left[ 1 + \left( \frac{F_{ci} / F_c'}{2c} \right) \right]^2 - \frac{F_{ci} / F_c'}{c}}$$

(3.7-1)

where:

- $F_c' = \text{reference compression design value parallel to grain multiplied by all applicable adjustment factors except } C_i (\text{see 2.3), psi}$
- $F_{ci} = \frac{0.822 \ E_{min}}{(\ell_e / d)^2}$
- $c = 0.8 \text{ for sawn lumber}$
- $c = 0.85 \text{ for round timber poles and piles}$
- $c = 0.9 \text{ for structural glued laminated timber or structural composite lumber}$
Analysis of Wood Columns

Data:
- Column – size, length
- Support conditions
- Material properties – $F_c$, $E$
- Load

Required:
- Pass/Fail or margin of safety

1. Calculate slenderness ratio $l_e/d$
   largest ratio governs. Must be < 50

2. Find adjustment factors
   $C_D$, $C_M$, $C_t$, $C_F$, $C_i$

3. Calculate $C_P$

4. Determine $F'c$ by multiplying the tabulated $F_c$
   by all the above factors

5. Calculate the actual stress: $f_c = P/A$

   $F'c > f_c$ passes
Analysis Example:

Data: section 4x8 (nominal)
Douglas Fir-Larch No1
M.C. 15%
P = 7000 LBS (Snow Load)

Find: Pass/Fail

From NDS Supplement Table 4A
Fc = 1500 psi
Emin = 620000 psi

\[
\begin{align*}
C_D &= 1.15 \\
C_M &= 1.0 \\
C_t &= 1.0 \\
C_F &= 1.05 \\
C_i &= 1.0 \\
C_P &= ? 
\end{align*}
\]

Analysis Example:

Calculate \( C_P \)

\[
C_P = \frac{1 + \frac{F_{ce} / F_c}{2c}}{\sqrt{\left[ 1 + \frac{F_{ce} / F_c}{2c} \right]^2 - \frac{F_{ce} / F_c}{c}}} 
\]

where:

\( F_c^* \) = reference compression design value parallel to grain multiplied by all applicable adjustment factors except \( C_s \) (see 2.3), psi

\[
F_{ce} = 0.822 E \left( \frac{\varepsilon_s}{d} \right) 
\]

\( c = 0.8 \) for sawn lumber
\( c = 0.85 \) for round timber poles and piles
\( c = 0.9 \) for structural glued laminated timber or structural composite lumber

\[
\begin{align*}
X - X &= \frac{300}{7.25} = 41.4 \\
Y - Y &= \frac{120}{3.5} = 34.3 \\
L_{ef} &= \frac{L_e}{d} = 41.4 < 50 \checkmark
\end{align*}
\]
Analysis Example:

Calculate $C_P$

\[
C_p = \frac{1 + \left( \frac{F_{c_e}}{F_c} \right)}{2c} - \sqrt{\left[ \frac{1 + \left( \frac{F_{c_e}}{F_c} \right)}{2c} \right]^2 - \frac{F_{c_e}}{F_c}} \quad (3.7-1)
\]

where:

- $F_c^*$ = reference compression design value parallel to grain multiplied by all applicable adjustment factors except $C_p$ (see 2.3), psi
- $F_{c_e} = \frac{0.822 \, E_{mn} \, \varepsilon}{(\varepsilon_y / d)^2}$
- $c = 0.8$ for sawn lumber
- $c = 0.85$ for round timber poles and piles
- $c = 0.9$ for structural glued laminated timber or structural composite lumber

\[
F_{c_e} = \frac{0.822 \, E_{mn} \, \varepsilon}{(\varepsilon_y / d)^2}
\]

\[
F_{c_e} = \frac{0.822 \, (200000)}{(41.4)^2} = 297.6 \text{ psi}
\]

\[
F_c^* = 1500 \left( 1.15 \right) = 1811.25 \text{ psi}
\]

\[
F_{c_e} / F_c^* = \frac{297.6}{1811.25} = 0.164
\]

\[
c = 0.8
\]

Analysis Example:

Calculate $C_P$

\[
C_p = \frac{1 + 0.164}{2(0.8)} - \sqrt{\left[ \frac{1 + 0.164}{2(0.8)} \right]^2 - 0.164}
\]

\[
C_p = 0.1584
\]

\[
F_c^* = 1500 \left( 1.15 \right) (1.05) (0.1584) = 286.9 \text{ psi}
\]

\[
F_c = \frac{P}{A} = \frac{7000}{25.38} = 275.8 \text{ psi}
\]

\[
F_c^* > F_c \quad \checkmark \text{ OK}
\]

Compare Allowable and Actual stress $F_c^* > F_c$ passes
Capacity Analysis of Columns

**Data:**
- Column – size, length
- Support conditions
- Material properties – $F_c$, $E$

**Required:**
- Maximum Load, $P_{\text{max}}$

1. Calculate slenderness ratio $l_e/d$
   - largest ratio governs. Must be < 50
2. Find adjustment factors
   - $C_D$, $C_M$, $C_t$, $C_F$, $C_i$
3. Calculate $C_P$
4. Determine $F'c$ by multiplying the tabulated $F_c$ by all the above factors
5. Set actual stress = allowable, $f_c = F'c$
6. Find the maximum allowable load $P_{\text{max}} = F'c \ A$

---

**Capacity Example**

**Data:**
- 4x10
- Hem – Fir, No 2 M.C. = 20%
- Wind Load
- $L_1 = 8'$, $L_2 = 4'$, $K_e = 1.0$

**Required:**
- Maximum Load, $P_{\text{max}}$

From NDS Supplement Table 4A
- $F_c = 1300$ psi
- $E_{\text{min}} = 470000$ psi

<table>
<thead>
<tr>
<th>$C_D$</th>
<th>$C_Mc$</th>
<th>$C_{ME}$</th>
<th>$C_t$</th>
<th>$C_F$</th>
<th>$C_i$</th>
<th>$C_P$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.6</td>
<td>0.8</td>
<td>0.9</td>
<td>1.0</td>
<td>1.0</td>
<td>1.0</td>
<td>?</td>
</tr>
</tbody>
</table>

$\frac{9}{9.25} = 10.41$
$\frac{48}{3.5} = 13.7$
$\frac{13.7}{10.41} < 1.3$
Allowable Flexure Stress $F_c'$

4 x 10

$F_c'$ from tables determined by species and grade

$F_c' = F_c(C_D C_M C_I C_F C_I C_P)$

Adjustment factors for compression:
- $C_M$ Moisture Factor
- $C_F$ Size Factor

### Wet Service Factor, $C_M$

When dimension lumber is used where moisture content will exceed 19% for an extended time period, design values shall be multiplied by the appropriate wet service factors from the following table:

<table>
<thead>
<tr>
<th>$F_S$</th>
<th>$F_I$</th>
<th>$F_V$</th>
<th>$F_{IC}$</th>
<th>$F_c'$</th>
<th>$F_t$</th>
<th>$E$ and $E_{min}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.85*</td>
<td>1.0</td>
<td>0.97</td>
<td>0.67</td>
<td>0.8**</td>
<td>0.9</td>
<td></td>
</tr>
</tbody>
</table>

* when $(F_c/C_t) \leq 1,150$ psi, $C_M = 1.0$

** when $(F_c/C_t) \leq 750$ psi, $C_M = 1.0$

### Capacity Example

Find $C_P$

\[
F_{ce} = 0.822 \frac{E_{min}}{(\epsilon_e/d)^2} = 0.822 \frac{470000(0.9)}{13.7^2} = 1848.7 \text{ psi}
\]

\[
F_{c} = 1300(1.6, 0.8)
\]

\[
F_{c}' = 1664.8 \text{ psi}
\]

\[
\frac{F_{ce}/F_{c}'}{F_{ce}/F_{c}} = 1.111
\]

\[
C_P = 0.7261
\]

Find the maximum load, $P_{max}$

\[
P_{max} = F_{c} A = 1208(32.38) = 39115 \text{ *}
\]
Stud Wall Design

**Given:**
- Lumber species, grade and size
- Conditions of use
- Load

**Required:**
- Stud spacing

1. Calculate slenderness ratio $\frac{l_o}{d}$
   largest ratio governs. Must be $< 50$

2. Find adjustment factors
   \[ C_D \ C_M \ C_t \ C_F \ C_i \]

3. Calculate $C_P$

4. Determine $F'_c$ by multiplying the tabulated $F_c$ by all the above factors

5. Set actual stress = allowable, $f_c = F'_c$

6. Find the capacity of one stud: $P_{\text{max}} = F'_c \ A$

7. Find allowable spacing (12", 16" or 24" o.c.)

8. Check bearing.

---

Stud Wall Example

**Data:**
- 2x6
- S-P-F, Stud M.C. = 12%
- D+L Load = 2500 PLF
- Braced as shown $K_e=1.0$

**Required:**
- o.c. spacing

From NDS Supplement Table 4A
- $F_c = 725$ psi
- $E_{\text{min}} = 440000$ psi

\[
\begin{align*}
C_D & = 1.0 \ (\text{LL}) \\
C_{Mc} & = 1.0 \quad C_{ME} = 1.0 \\
C_t & = 1.0 \\
C_F & = 1.0 \ (\text{stud}) \\
C_i & = 1.0 \\
C_P & = ? 
\end{align*}
\]
Stud Wall Example

\[ S - P - F \text{ STUD GRADE} \]
\[ E_c = 725 \text{ psi} \]
\[ E_{min} = 440,000 \text{ psi} \]
\[ C_D = 1 \quad C_M = 1 \quad C_p = 10 \]

\[ x - x \]
\[ L_x = 124.5" \]
\[ \frac{L_x}{d} = 22.6 \]

\[ y - y \]
\[ L_y = 40" \]
\[ \frac{L_y}{d} = 26.7 \]

\[ P_D = 26.7 \]

\[ F_{CE} = \frac{0.822 \cdot E_{min}}{(C_p)^2} = \frac{0.822 \cdot 440,000}{26.7^2} = 508.6 \]

\[ F_c = 725 (1\times1\times1) = 725 \text{ psi} \]

\[ F_{p_c} = \frac{508.6}{725} = 0.702 \]

NDS eq. 3.7-1 \[ \rightarrow C_p = 0.559 \]
Stud Wall Example

Find max allowable stress, $F'_c$

\[
F'_c = 725 \left(0.559\right) = 405.6 \text{ psi}
\]

Calculate max load per stud

\[
P = F'_c A = 405.6 \text{ psi} \times 8.25 \text{ in}^2 = 3345 \text{ lb}
\]

Determine max stud spacing

\[
\frac{2500 \text{ plf}}{3345 \text{ lbs/stud}} \times \frac{12}{8} = 16" \text{ o.c. (rounded down)}
\]

Stud Wall Example

Check bearing on sill plate

### 3.10.4 Bearing Area Factor, $C_b$

Reference compression design values perpendicular to grain, $F_{cm}$, apply to bearings of any length at the ends of a member, and to all bearings 6" or more in length at any other location. For bearings less than 6" in length and not nearer than 3” to the end of a member, the reference compression design value perpendicular to grain, $F_{cm}$, shall be permitted to be multiplied by the following bearing area factor, $C_b$:

\[
C_b = \frac{\ell_b + 0.3 / 5}{\ell_b}
\]

where:

$\ell_b = $ bearing length measured parallel to grain, in.

Equation 3.10-2 gives the following bearing area factors, $C_b$, for the indicated bearing length on such small areas as plates and washers:

<table>
<thead>
<tr>
<th>$\ell_b$</th>
<th>0.5&quot;</th>
<th>1&quot;</th>
<th>1.5&quot;</th>
<th>2&quot;</th>
<th>3&quot;</th>
<th>4&quot;</th>
<th>6&quot; or more</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_b$</td>
<td>1.75</td>
<td>1.38</td>
<td>1.25</td>
<td>1.19</td>
<td>1.13</td>
<td>1.10</td>
<td>1.00</td>
</tr>
</tbody>
</table>

For round bearing areas such as washers, the bearing length, $\ell_b$, shall be equal to the diameter.
Stud Wall Example

Check bearing on sill plate

\[
\text{BEARING (on SILL/PLATE)}
\]

\[
1.5 = b
\]
\[
C_b = 1.25
\]

\[
\text{MINIMUM } F_{Cl} = 425 \quad F_{Cl} = 425(1.25) = 531.25 \text{ psi}
\]

\[
\text{ACTUAL } F_{Cl} = \frac{P}{A} = \frac{3333}{8.25} = 404.0 \text{ psi} <
\]

\[
\text{OK}
\]